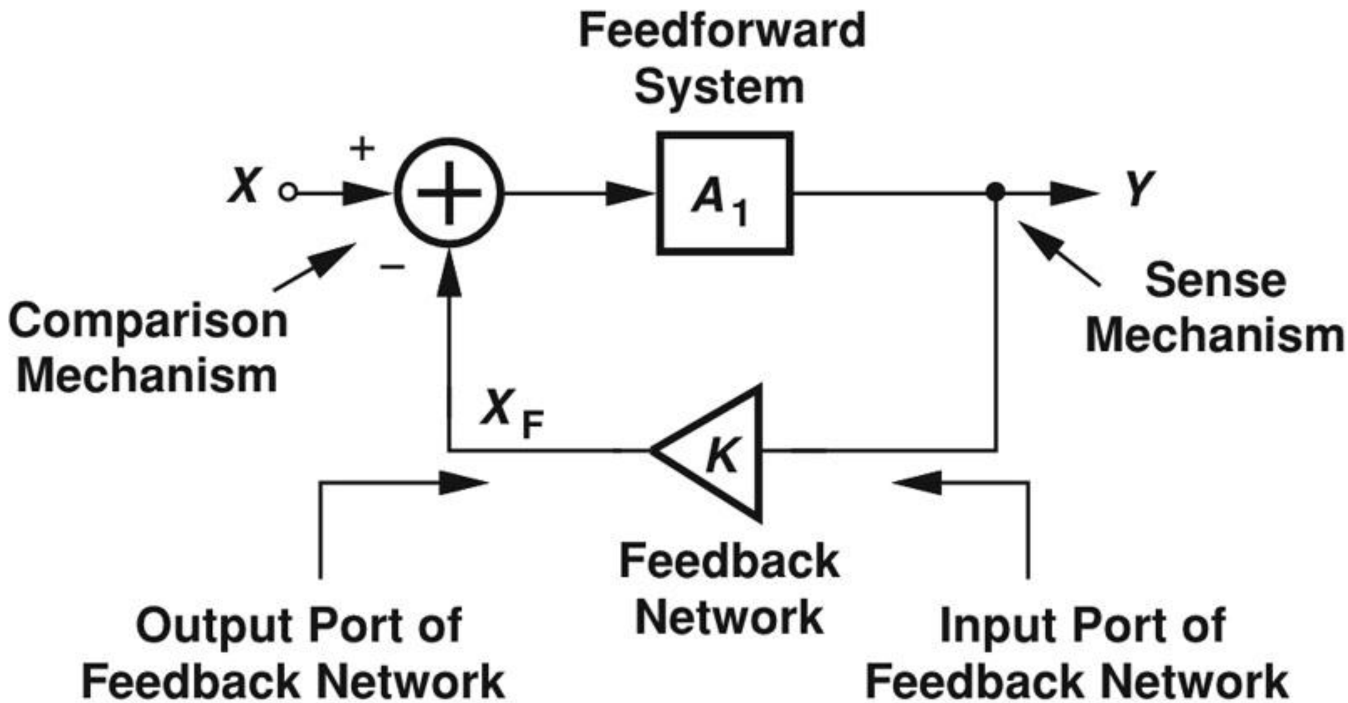
CHAPTER 7

Feedback, Stability, Compensation

Syllabus

- Basic physics of MOS transistors and their equivalent circuit models (ch. 6)
- Elementary gain stages (ch. 7, ch. 9, and ch. 10)
- Operational amplifier basics (ch. 8)
- Frequency responses (ch. 11)
- Noise (ch.7, Design of analog CMOS ICs)
- Feedback, stability and compensation (ch. 12)
- Operational amplifiers (ch.9, Design of analog CMOS ICs)

Negative Feedback System



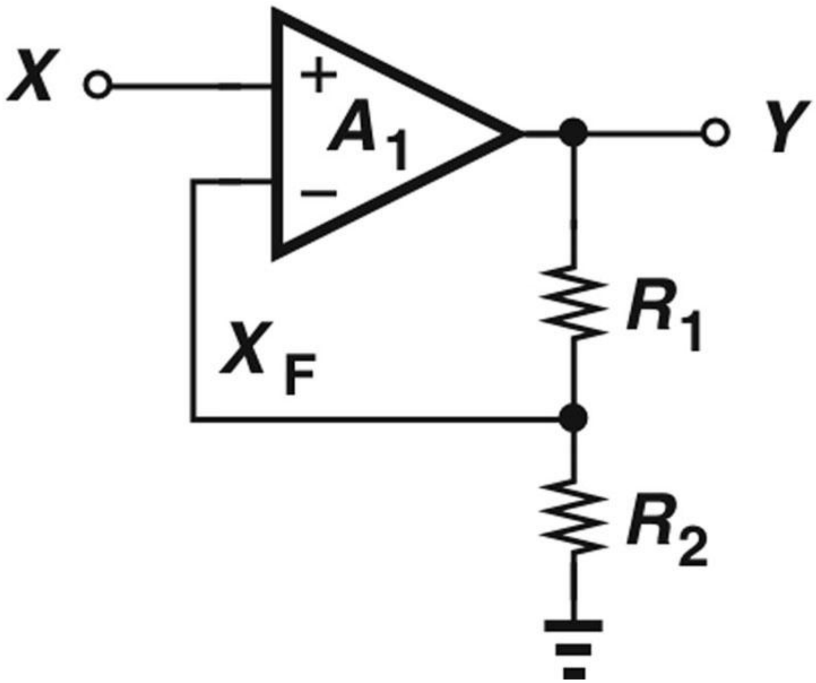
$$(X - KY)A_1 = Y.$$

$$\boxed{\frac{Y}{X} = \frac{A_1}{1 + KA_1}}$$

Closed-loop gain

- **A negative feedback system consists of four components:**
- **1) feedforward system, 2) sense mechanism, 3) feedback network, and 4) comparison mechanism.**

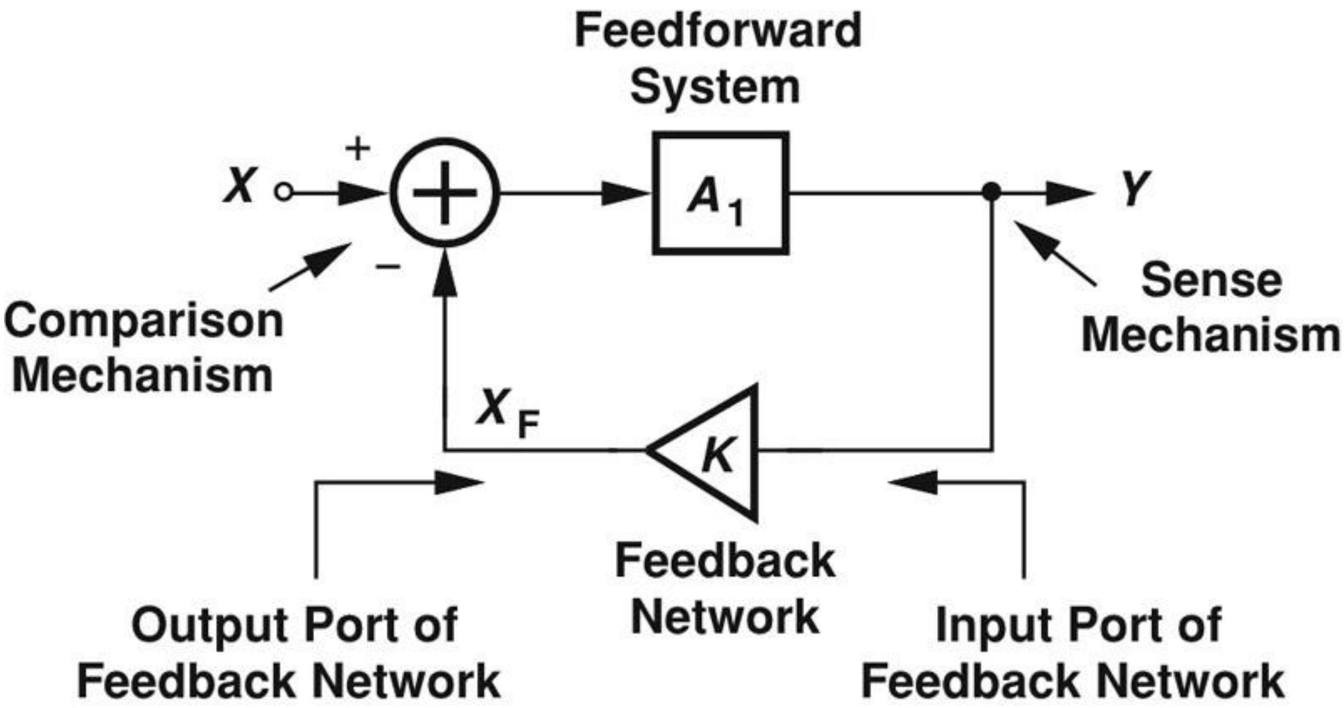
Example: Noninverting Amplifier



$$\frac{Y}{X} = \frac{A_1}{1 + \frac{R_2}{R_1 + R_2} A_1}$$

➢ **A_1 is the feedforward network, R_1 and R_2 provide the sensing and feedback capabilities, and comparison is provided by differential input of A_1.**

Comparison Error



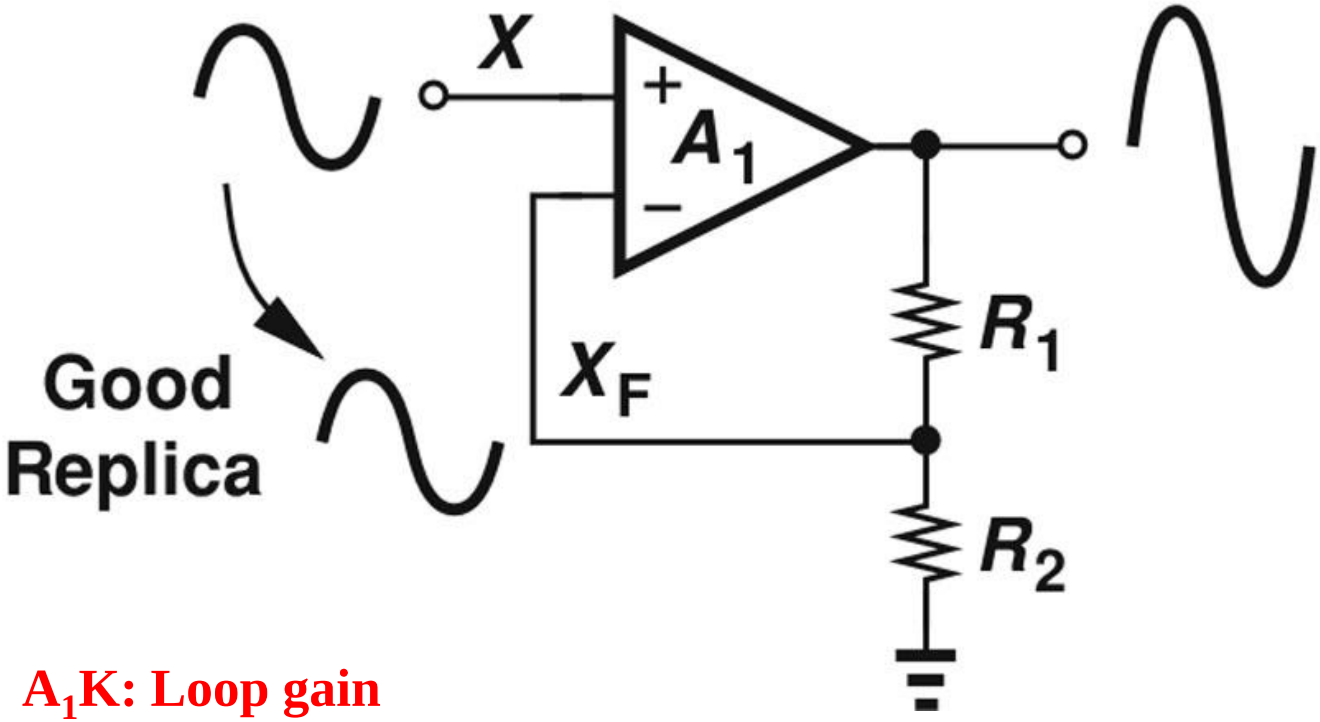
$$E = X - X_F$$

$$X_F = KA_1E$$

$$\boxed{E = \frac{X}{1 + A_1K}}$$

- **As A_1K increases, the error between the input and fed back signal decreases. Or the fed back signal approaches a good replica of the input.**

A Good Replica

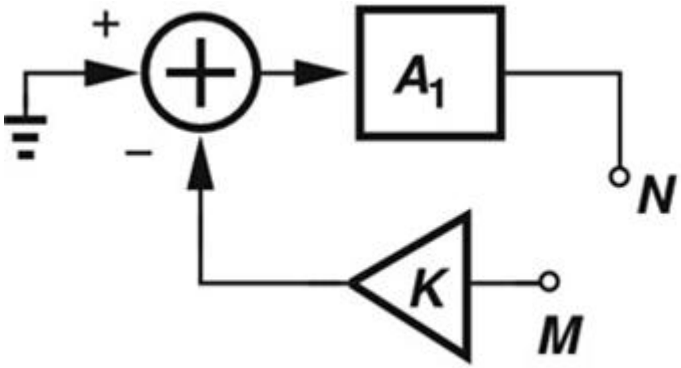


A_1K: Loop gain

- As A_1K increases, the error between the input and fed back signal decreases. Or the fed back signal approaches a good replica of the input.

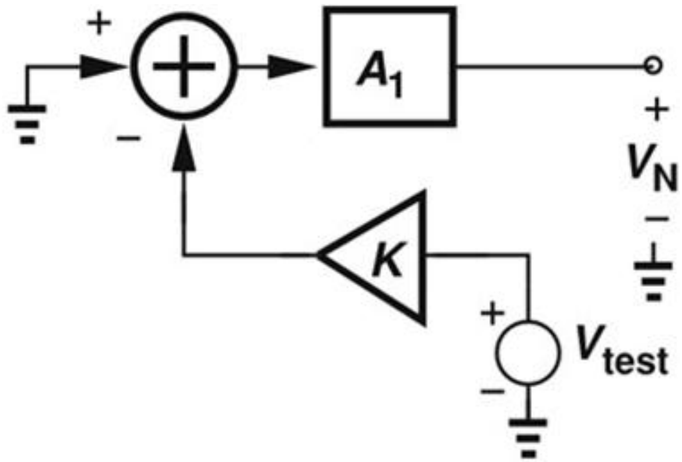
Loop Gain

Set the input to zero and "break" the loop at an arbitrary point



A system with an input M and an output N

Apply V_{test} at the input

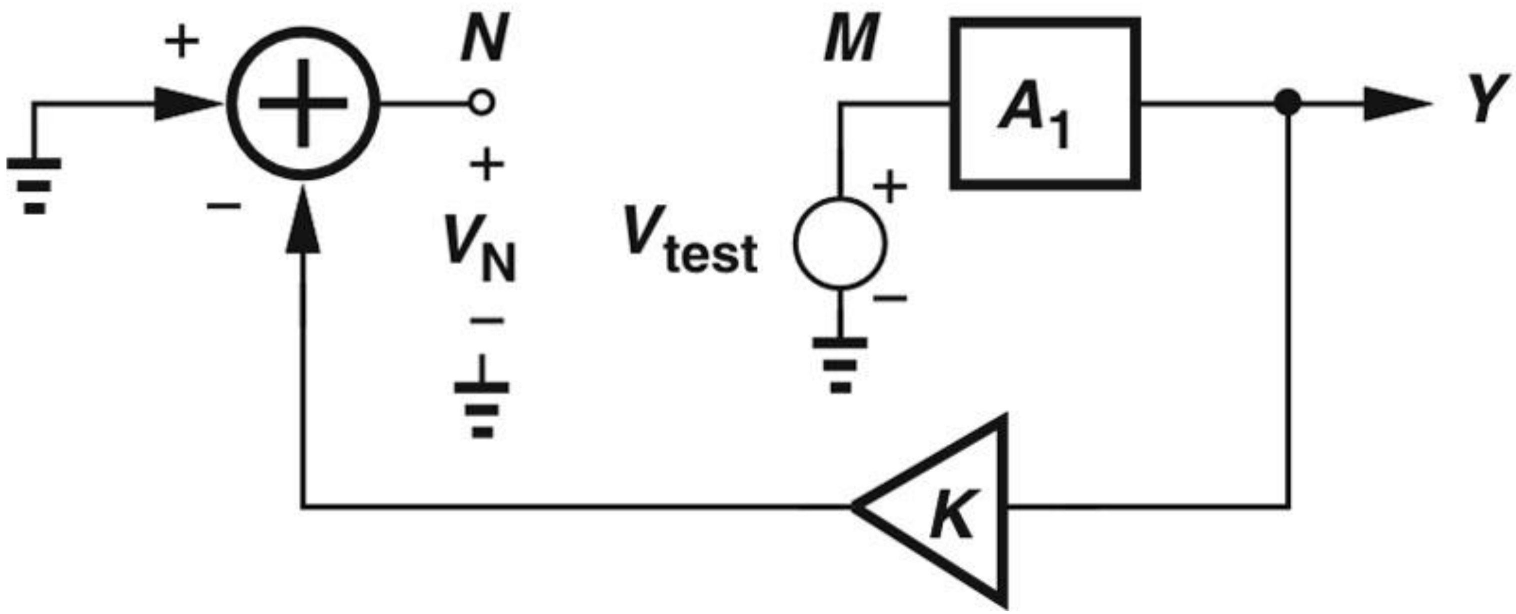


$$V_N = -KV_{test}A_1$$

$$\Rightarrow KA_1 = -\frac{V_N}{V_{test}}$$ **Loop gain**

$$\frac{Y}{X} = \frac{A_1}{1 + KA_1}$$ **Closed-loop gain**

Independent of Breaking Point



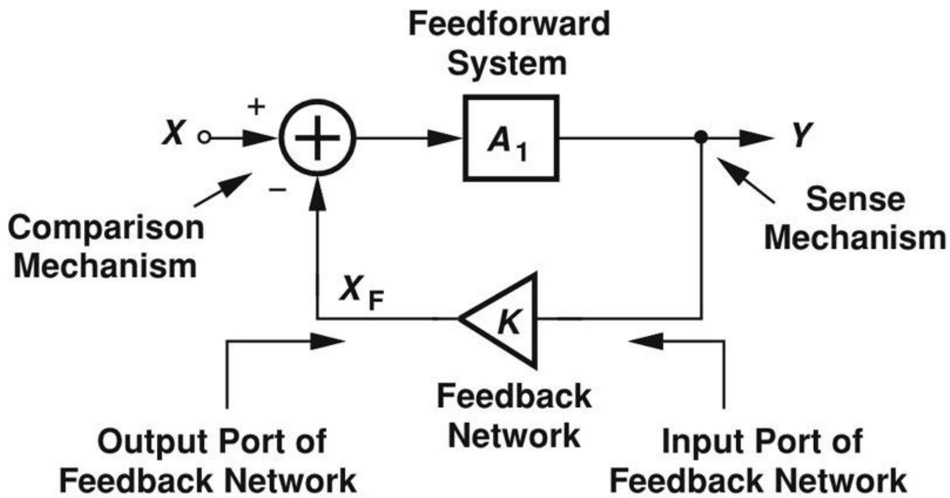
$$V_N = -KA_1V_{test}$$

➢ **Breaking the loop at another point yields the same loop gain.**

Merits of Negative Feedback

- Gain desensitization against open-loop gain variations due to process and temperature variations.
- Increase the bandwidth of the circuit
- Insensitive to load variations
- Improve the linearity of the circuit

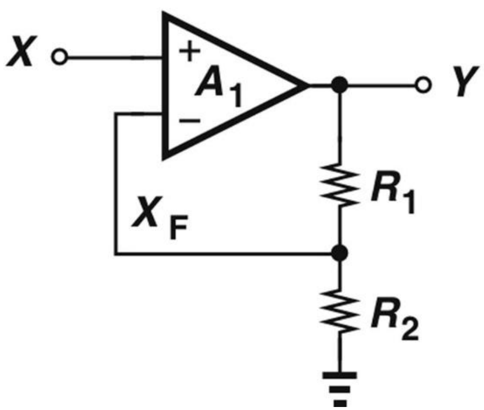
Gain Desensitization



$$\frac{Y}{X} = \frac{A_1}{1 + KA_1}$$

$$A_1 K >> 1 \Rightarrow \boxed{\frac{Y}{X} \approx \frac{1}{K}}$$

Noninverting amplifier



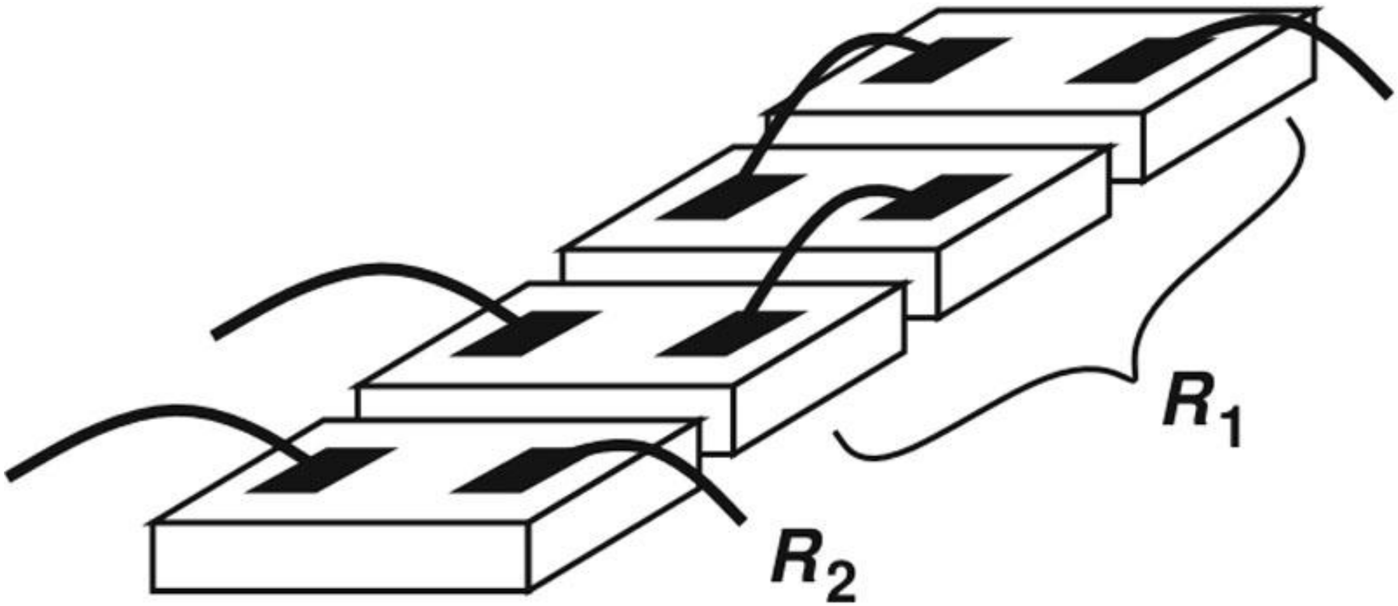
$$\frac{Y}{X} = \frac{A_1}{1 + \frac{R_2}{R_1 + R_2} A_1}$$

$$A_1 R_2/(R_1 + R_2) \gg 1$$

$$\Rightarrow \frac{Y}{X} \approx \frac{1}{K} \approx 1 + \frac{R_1}{R_2}$$

- A large loop gain is needed to create a precise gain, one that does not depend on A_1, which can vary by ±20%.

Matching by Resistor Ratios



- When two resistors are composed of the same unit resistor, their ratio is very accurate. Since when they vary, they will vary together and maintain a constant ratio.

Bandwidth Extension

One-pole open-loop amplifier

$$\frac{Y}{X} = \frac{A_1}{1 + KA_1} \qquad A_1(s) = \frac{A_0}{1 + \dfrac{s}{\omega_0}} \qquad \Rightarrow \frac{Y}{X} = \frac{\dfrac{A_0}{1 + \dfrac{s}{\omega_0}}}{1 + K\dfrac{A_0}{1 + \dfrac{s}{\omega_0}}}$$

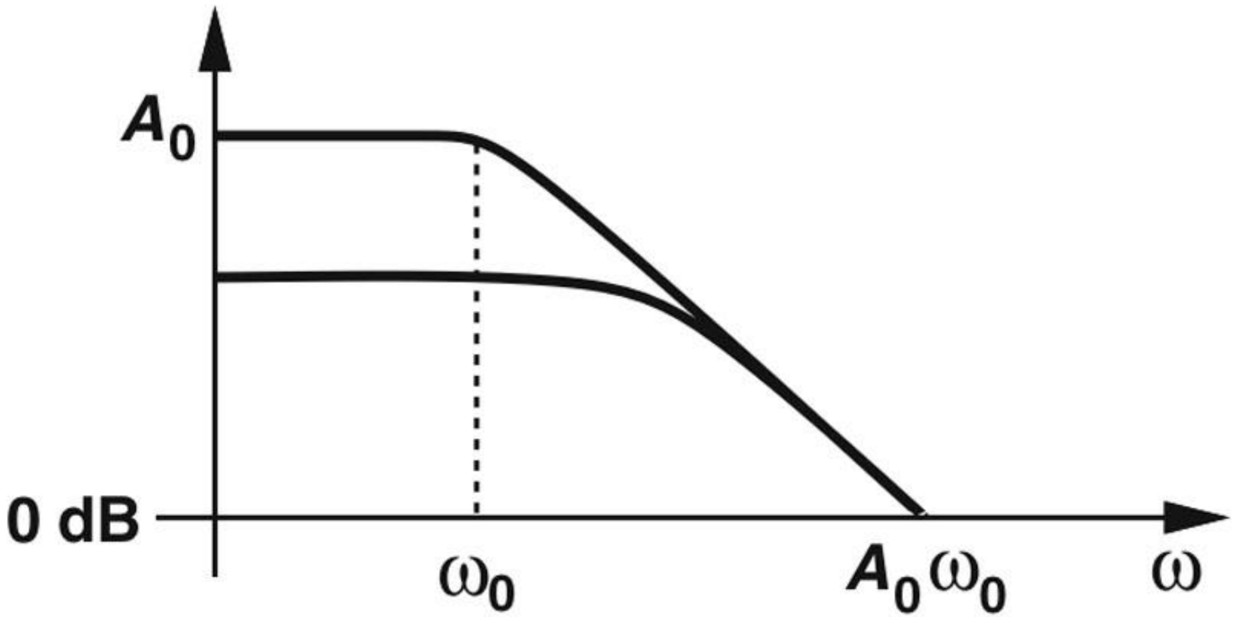
$$\Rightarrow \frac{Y}{X}(s) = \frac{A_0}{1 + KA_0 + \dfrac{s}{\omega_0}} = \frac{\dfrac{A_0}{1 + KA_0}}{1 + \dfrac{s}{(1 + KA_0)\omega_0}}$$

$$\text{Closed-Loop Gain} = \frac{A_0}{1 + KA_0}$$

$$\text{Closed-Loop Bandwidth} = (1 + KA_0)\omega_0$$

- **The gain and bandwidth are scaled by the same factor but opposite directions, displaying a *constant* product.**

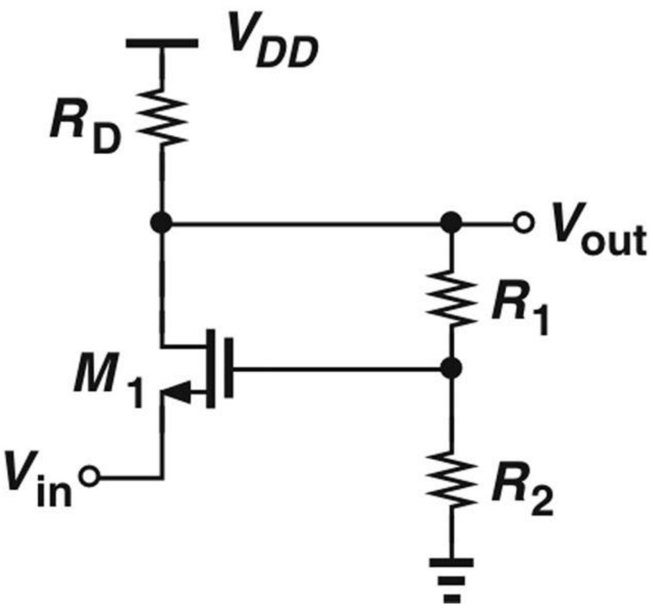
Bandwidth Extension with Loop Gain



- **As the loop gain increases, we can see the decrease of the overall gain and the extension of the bandwidth.**
- **The unity-gain bandwidth remains independent with loop gain.**

Modification of I/O Impedances

<Example 12.7> Assume λ=0 and $R_1 + R_2 >> R_D$ for simplicity. (a) Identify the four components of the feedback system.



1. **Forward system: M_1 and R_D, Common-gate**
2. **Sense and feedback: R_1 and R_2**
3. **Subtractor input from feedback** $V_{out}R_2/(R_1+R_2)$

Transistor M1 operates as a the subtractor because the small-signal drain current is proportional to the difference between the gate and source voltages $i_D = g_m(v_G - v_S)$

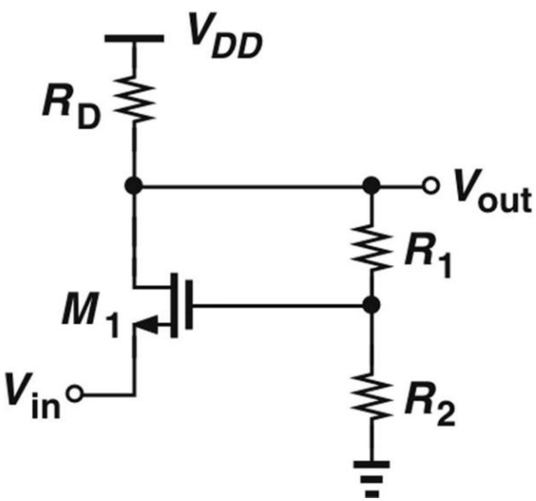
(b) Determine the open-loop and closed-loop voltage gain.

$$A_0 \approx g_m R_D$$

$$\frac{v_{out}}{v_{in}} = \frac{A_0}{1+KA_0} = \frac{g_m R_D}{1+\dfrac{R_2}{R_1+R_2}g_m R_D}$$

Modification of I/O Impedances

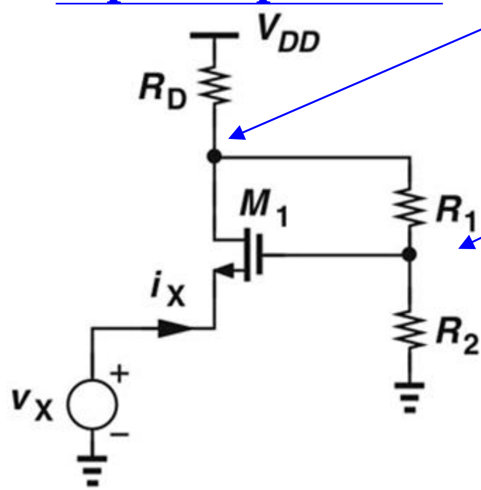
<Example 12.7> Assume λ=0 and $R_1 + R_2 >> R_D$ for simplicity. (c) Determine the open-loop and closed-loop I/O impedances.



$$R_{in,open} = \frac{1}{g_m}$$

$$R_{out,open} = R_D$$

Input impedance



$R_1 + R_2 >> R_D$, $v_{D1} = i_x R_D$

$$+i_X R_D R_2/(R_1+R_2)$$

$$\Rightarrow i_D = g_m v_{GS}$$

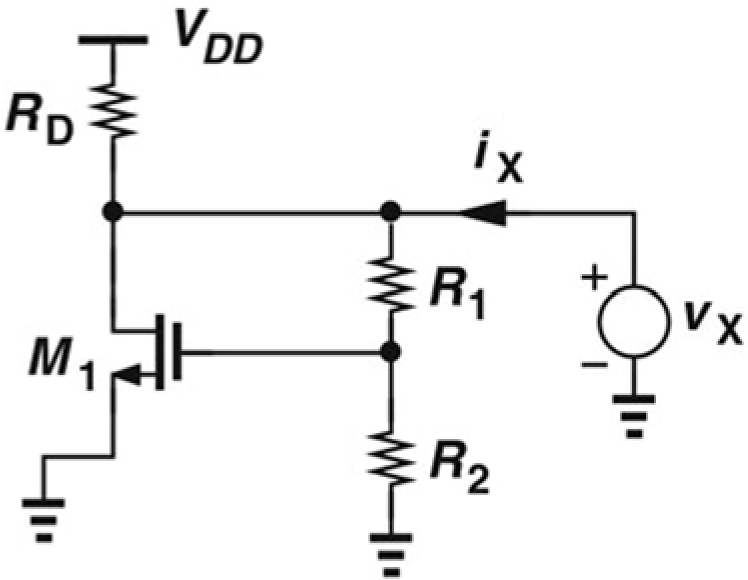
$$= g_m\left(\frac{+i_X R_D R_2}{R_1+R_2} - v_X\right)$$

$$i_D = -i_X \quad \Rightarrow \quad \frac{v_X}{i_X} = \frac{1}{g_m}\underbrace{\left(1 + \frac{R_2}{R_1+R_2} g_m R_D\right)}_{1+KA_0}$$

- **The input resistance *increases* by the same factor which the gain decreases**

Modification of I/O Impedances

Output impedance



$$v_{GS} = \frac{R_2}{R_1 + R_2} v_X$$

$$i_D = g_m v_{GS} = g_m \frac{R_2}{R_1 + R_2} v_X$$

if $R_1 + R_2 \gg R_D$, then $i_X \approx i_D + v_X / R_D$

➡ $$i_X \approx g_m \frac{R_2}{R_1 + R_2} v_X + \frac{v_X}{R_D}$$

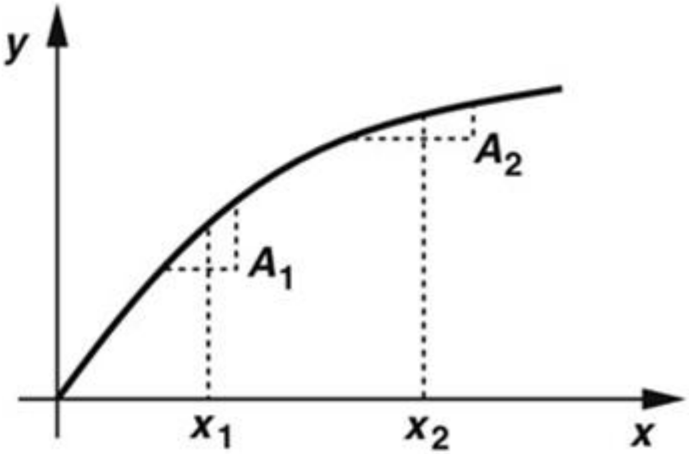
➡ $$\frac{v_X}{i_X} = \frac{R_D}{\underbrace{1 + \frac{R_2}{R_1 + R_2} g_m R_D}_{1+KA_0}}$$

1. Does Rin always increase?
2. Does Rout always decrease?
3. Does feedback always desirable?

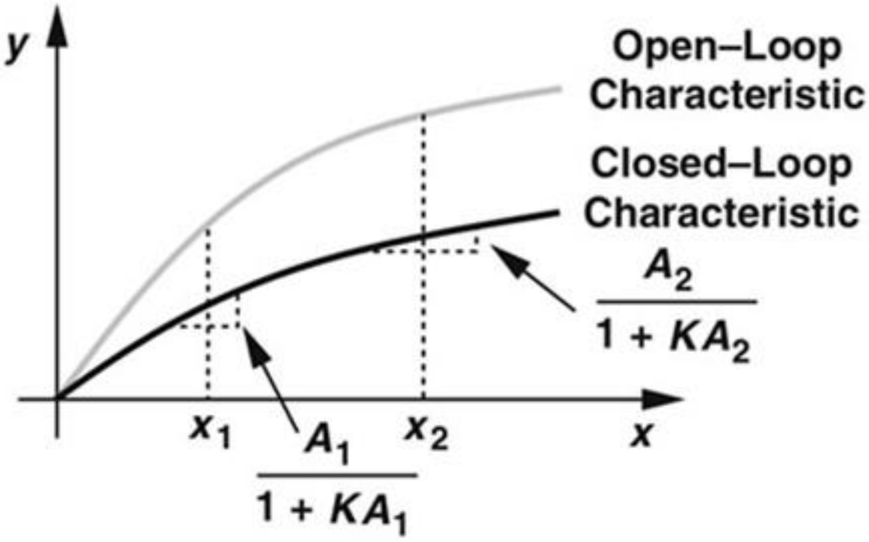
➢ **The output resistance *decreases* by the same factor which the gain decreases**

Linearity Improvement

Nonlinear gain characteristics



Nonlinear gain with feedback

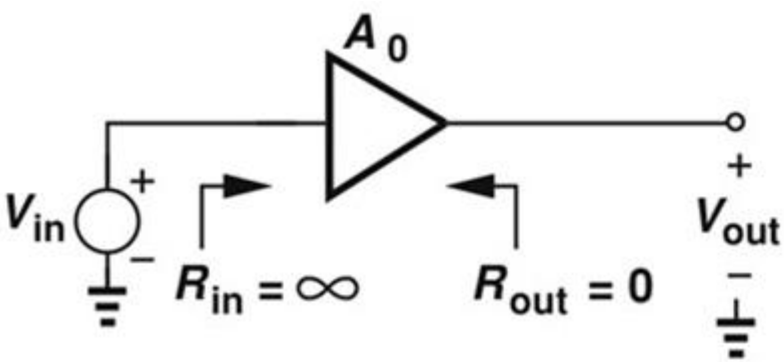


$$\text{Gain at } x_1 = \frac{A_1}{1+KA_1} \approx \frac{1}{K}\left(1 - \frac{1}{KA_1}\right) \quad \text{Gain at } x_2 = \frac{A_2}{1+KA_2} \approx \frac{1}{K}\left(1 - \frac{1}{KA_2}\right)$$

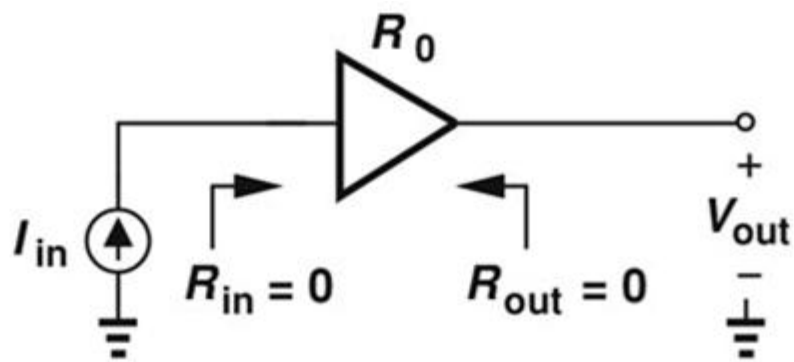
- **As long as KA_1 and KA_2 are large, the variation of the closed-loop gain with the signal level remains much less than that of the open-loop gain.**

Four Types of Amplifiers

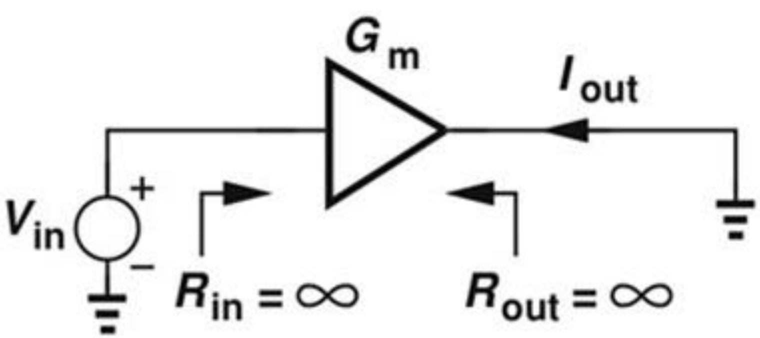
<Voltage amplifier>



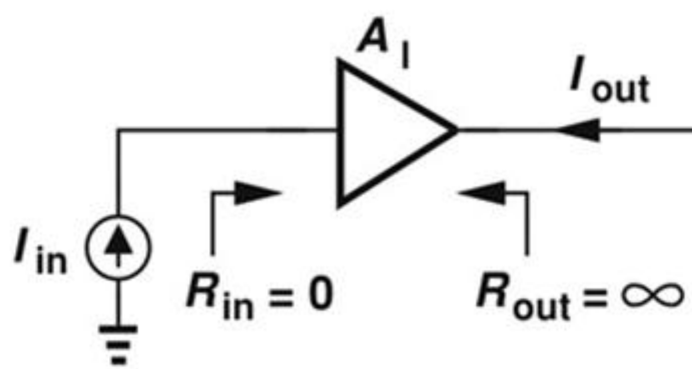
<Transimpedance amplifier>



<Transconductance amplifier>

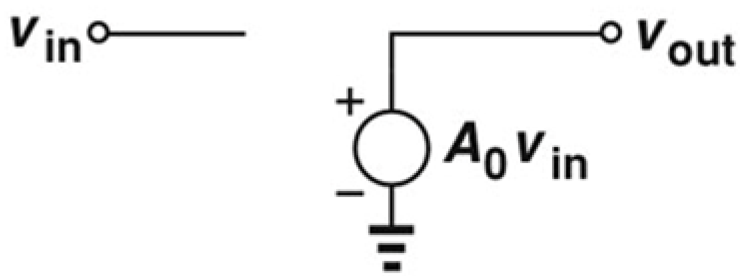


<Current amplifier>



Ideal Models of the Four Amplifiers

<Voltage amplifier>



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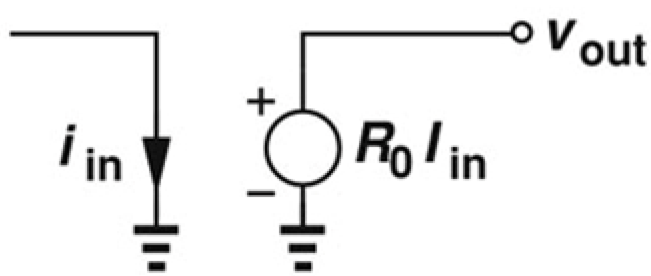

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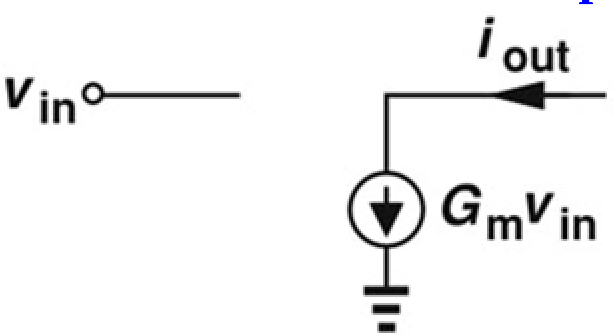
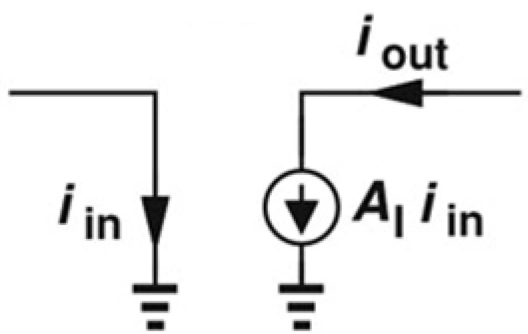
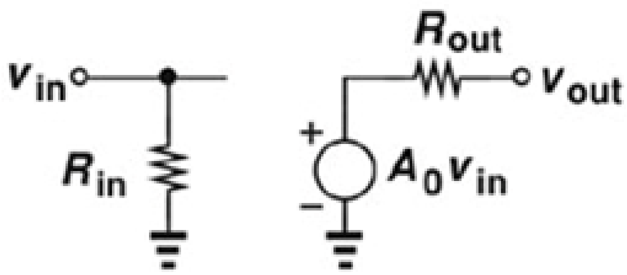


<Current amplifier>

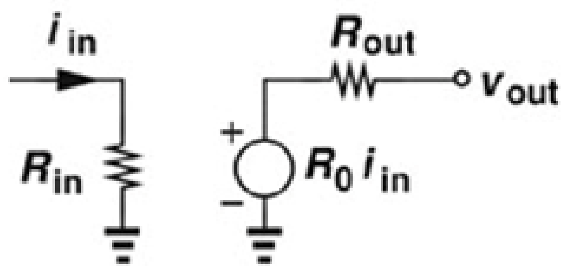


Realistic Models of the Four Amplifiers

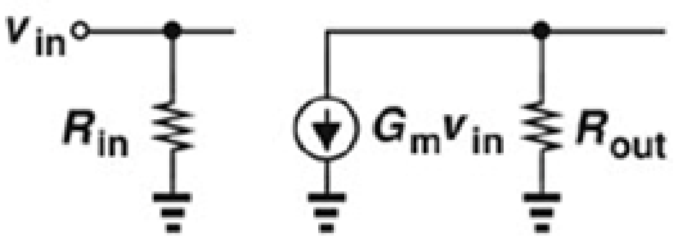
<Voltage amplifier>



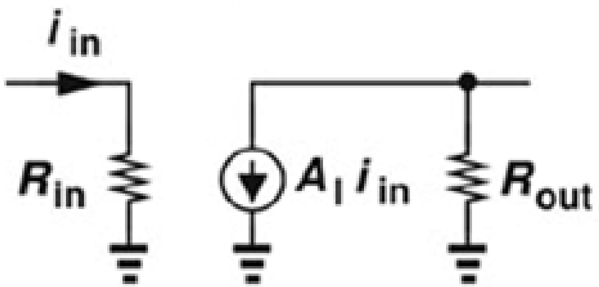
<Transimpedance amplifier>



<Transconductance amplifier>

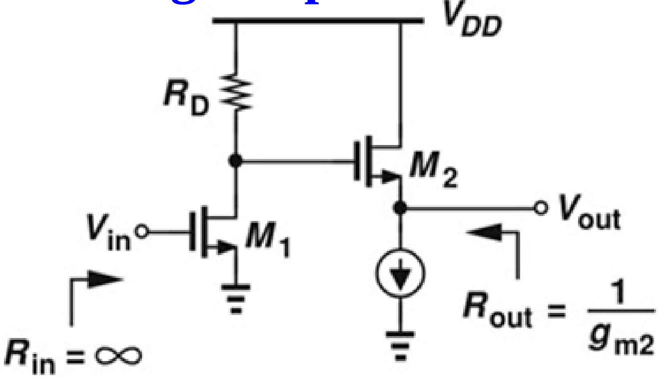


<Current amplifier>

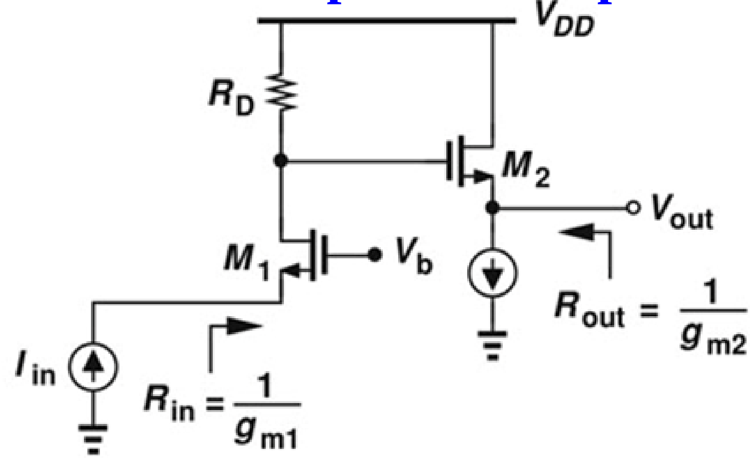


Examples of the Four Amplifiers

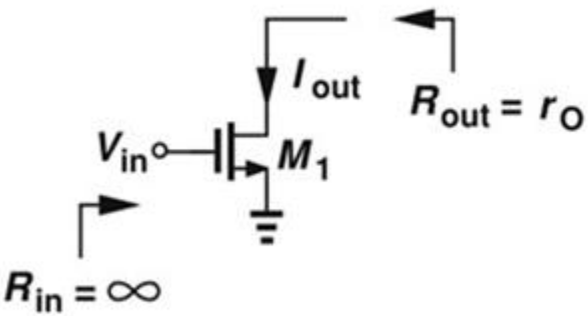
<Voltage amplifier>



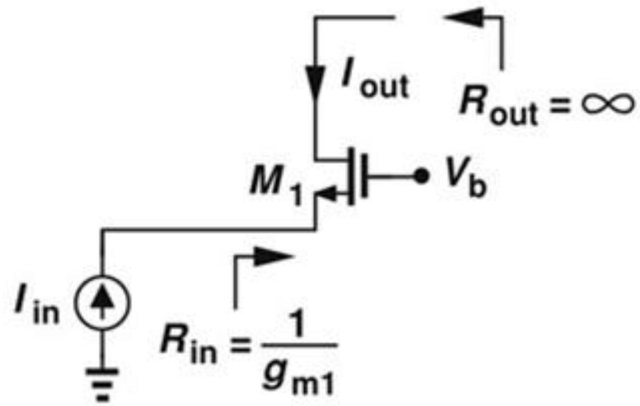
<Transimpedance amplifier>



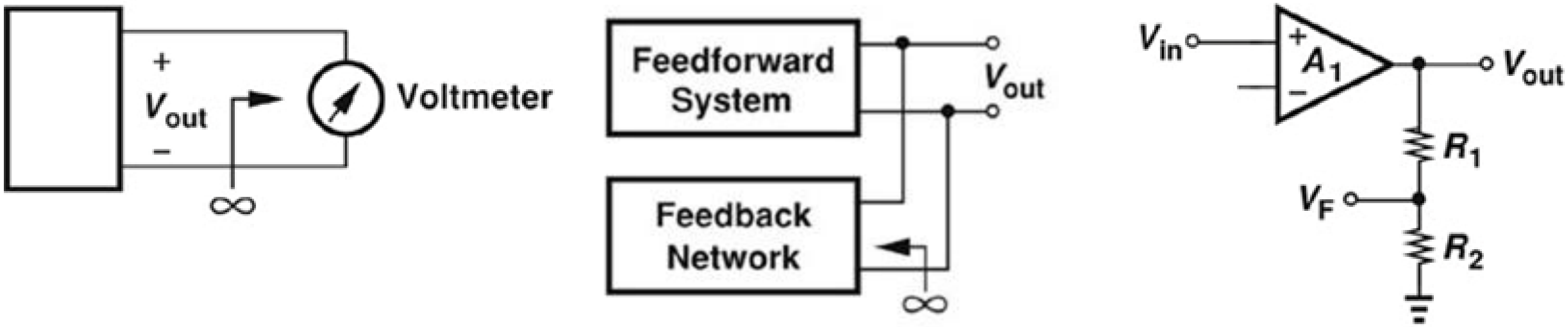
<Transconductance amplifier>



<Current amplifier>



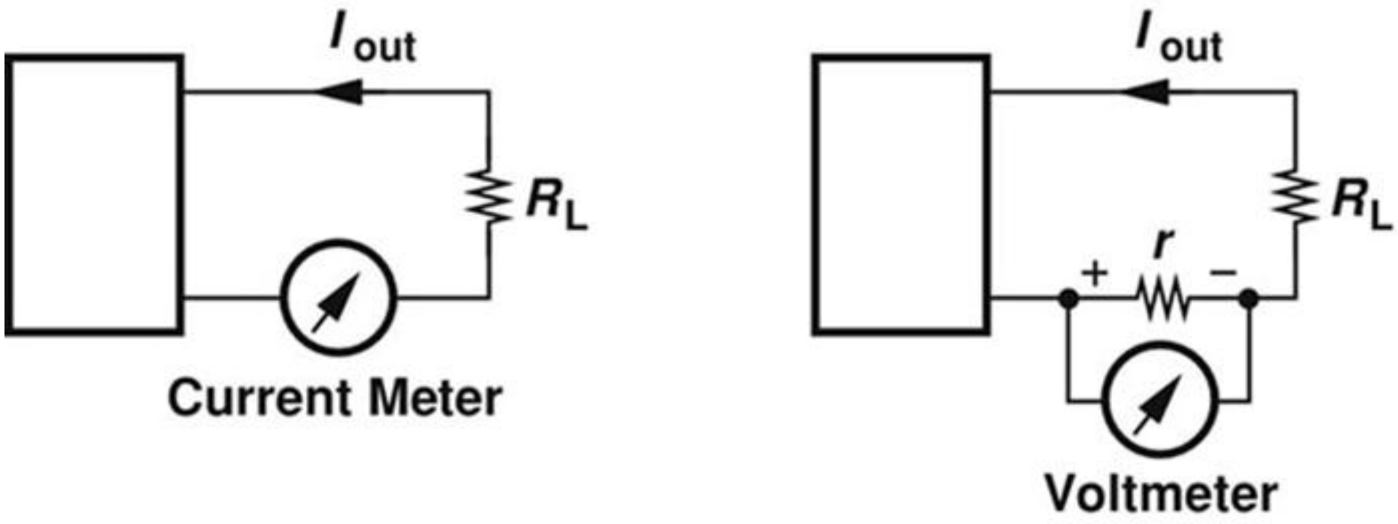
Sensing and Returning a Voltage



$$R_1 + R_2 \approx \infty$$

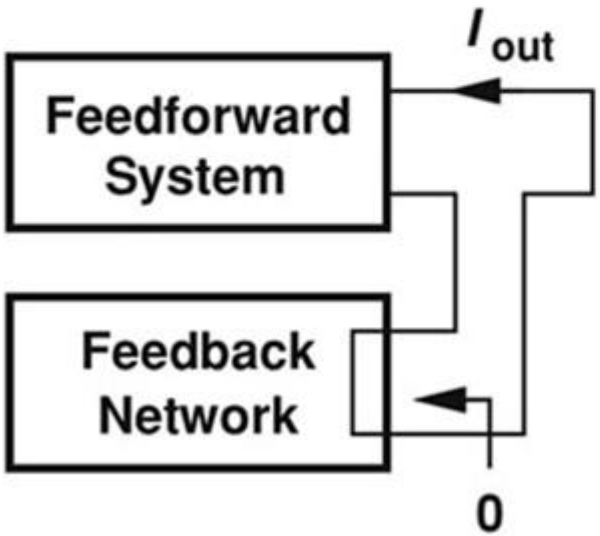
- **In order to sense a voltage across two terminals, a voltmeter with ideally infinite impedance is used.**
- **Similarly, for a feedback network to correctly sense the output voltage, its input impedance needs to be large.**
- **R_1 and R_2 also provide a mean to return the voltage.**

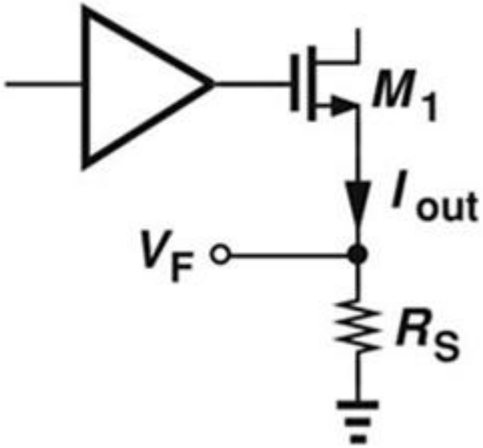
Sensing a Current



- **A current is measured by inserting a current meter with ideally zero impedance in series with the conduction path.**
- **The current meter is composed of a small resistance r in parallel with a voltmeter.**

Sensing and Returning a Current



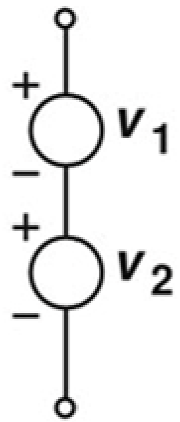


$$R_S \approx 0$$

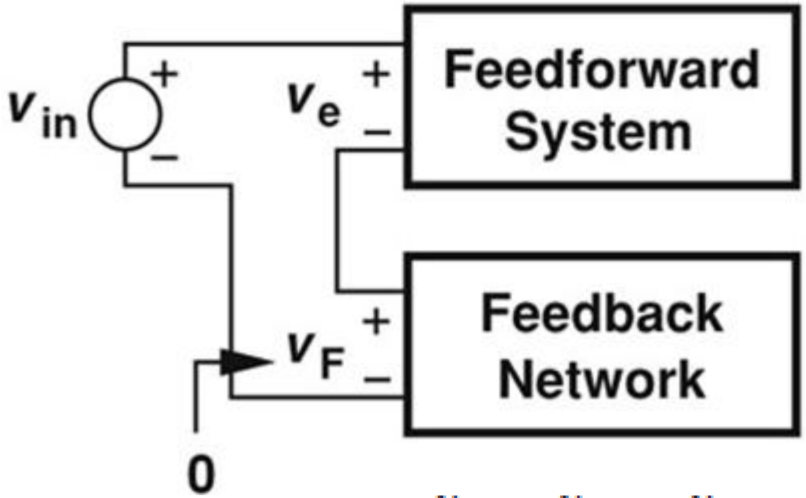
- **Similarly for a feedback network to correctly sense the current, its input impedance has to be small.**
- **R_S has to be small so that its voltage drop will not change I_{out}.**

Addition of Two Voltage Sources

Addition of two voltages



Addition of feedback and input voltages

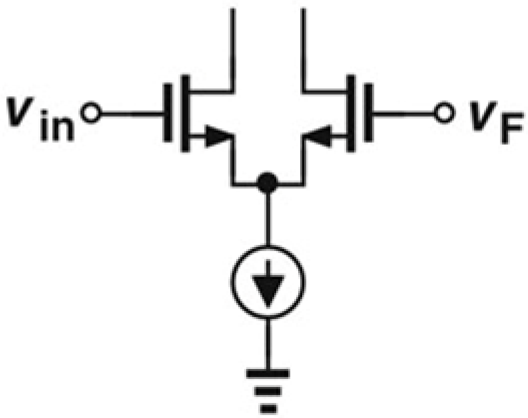


$$v_e = v_{in} - v_F$$

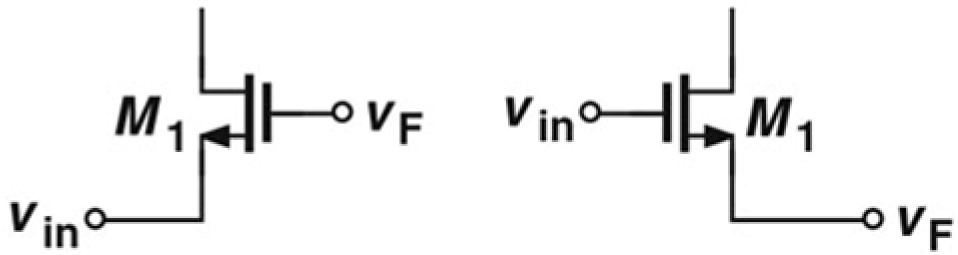
- **In order to add or substrate two voltage sources, we place them in series. So the feedback network is placed in series with the input source.**

Practical Circuits to Subtract Voltages

Differential pair as voltage subtractor



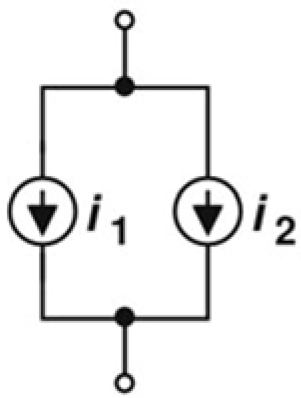
Single transistor as a voltage subtractor



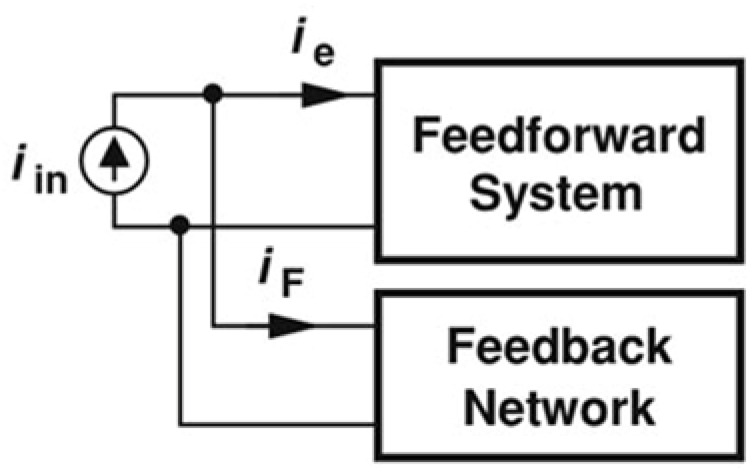
- **Although not directly in series, V_{in} and V_F are being subtracted since the resultant currents, differential and single-ended, are proportional to the difference of V_{in} and V_F.**

Addition of Two Current Sources

Addition of two currents



Addition of feedback and input currents

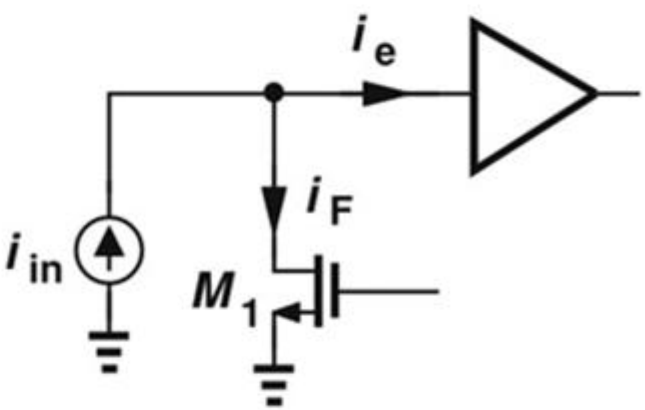


$$i_e = i_{in} - i_F$$

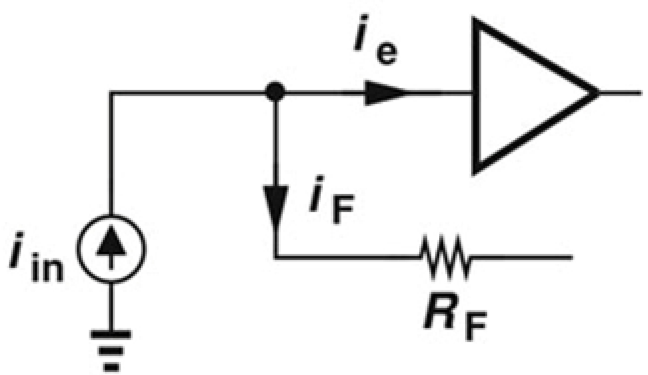
- In order to add two current sources, we place them in parallel. So the feedback network is placed in parallel with the input signal.

Practical Circuits to Subtract Currents

Circuit realization #1

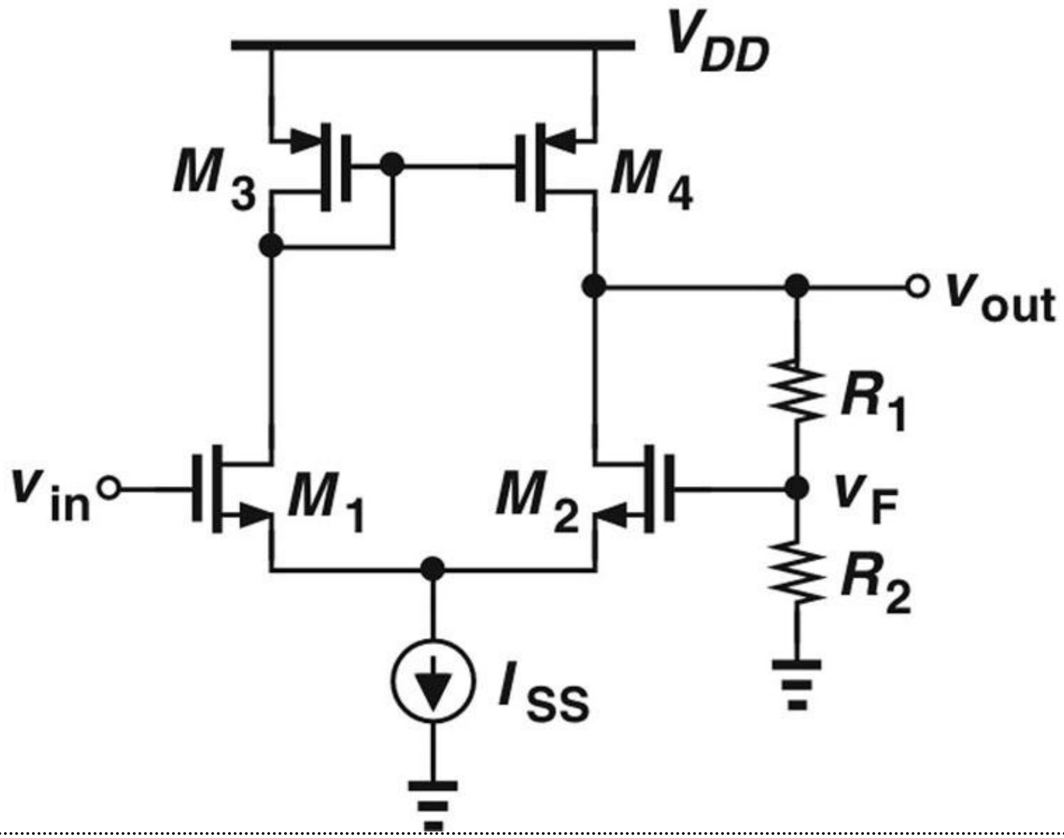


Circuit realization #2



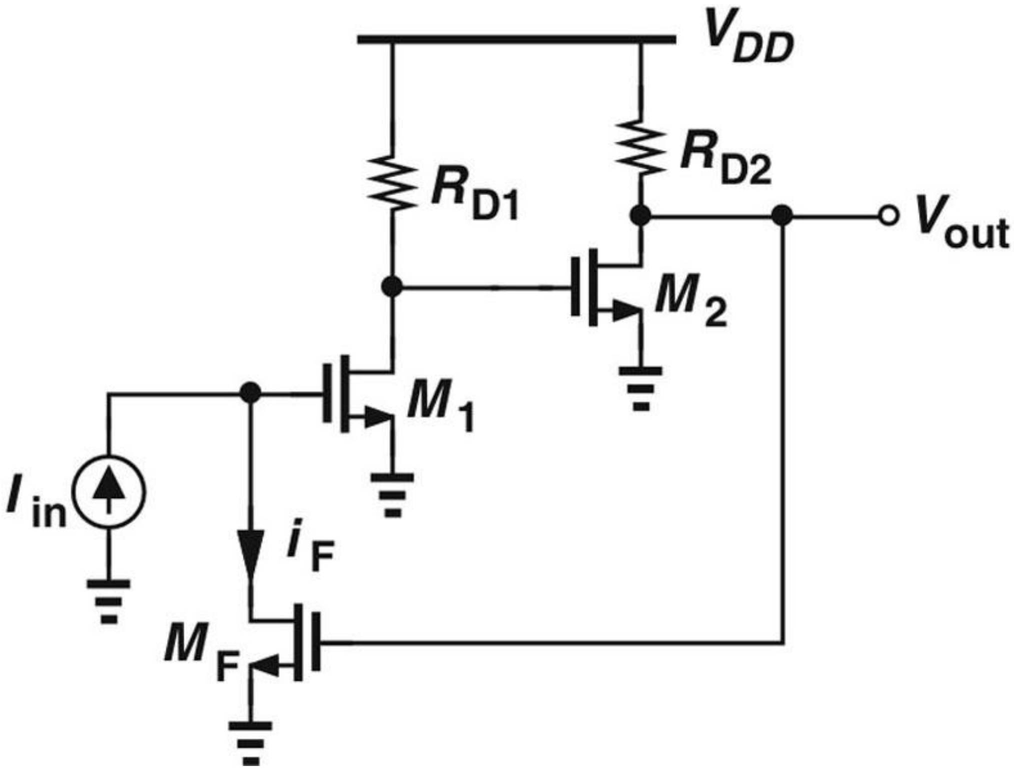
- **Since M_1 and R_F are in parallel with the input current source, their respective currents are being subtracted. Note, R_F has to be large enough to approximate a current source.**

Example: Sense and Return



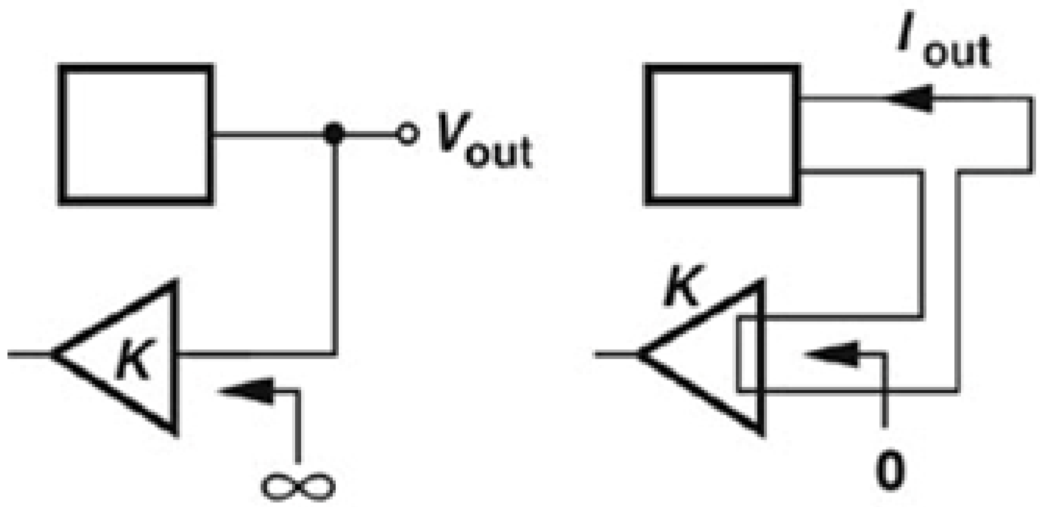
- R_1 and R_2 sense and return the output voltage to feedforward network consisting of M_1- M_4.
- M_1 and M_2 also act as a voltage subtractor.

Example: Sense and Return



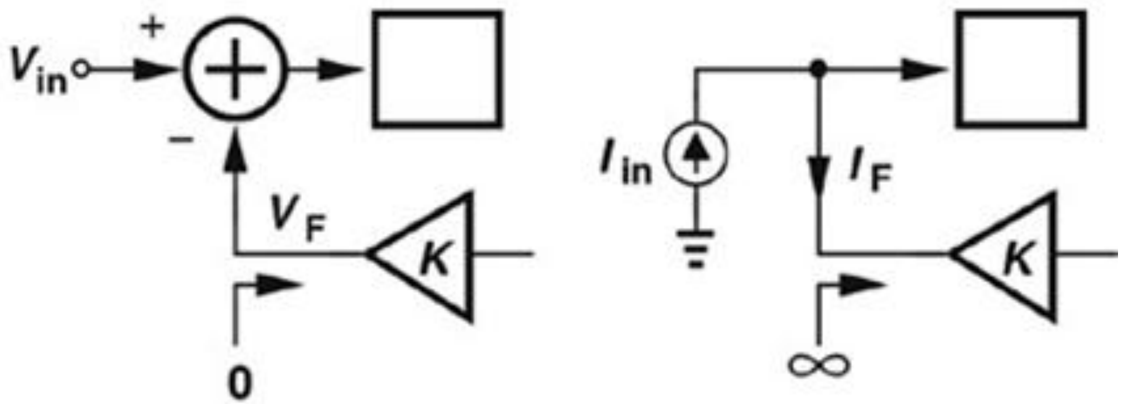
- M_F senses the output voltage and returns a current to the input.
- Feedback factor is given by $K = i_F/v_{out} = g_{mF}$.

Input Impedance of Ideal FB Network



- To sense a voltage, the input impedance of an ideal feedback network must be infinite.
- To sense a current, the input impedance of an ideal feedback network must be zero.

Output Impedance of Ideal FB Network



- To return a voltage, the output impedance of an ideal feedback network must be zero.
- To return a current, the output impedance of an ideal feedback network must be infinite.

Polarity of Feedback

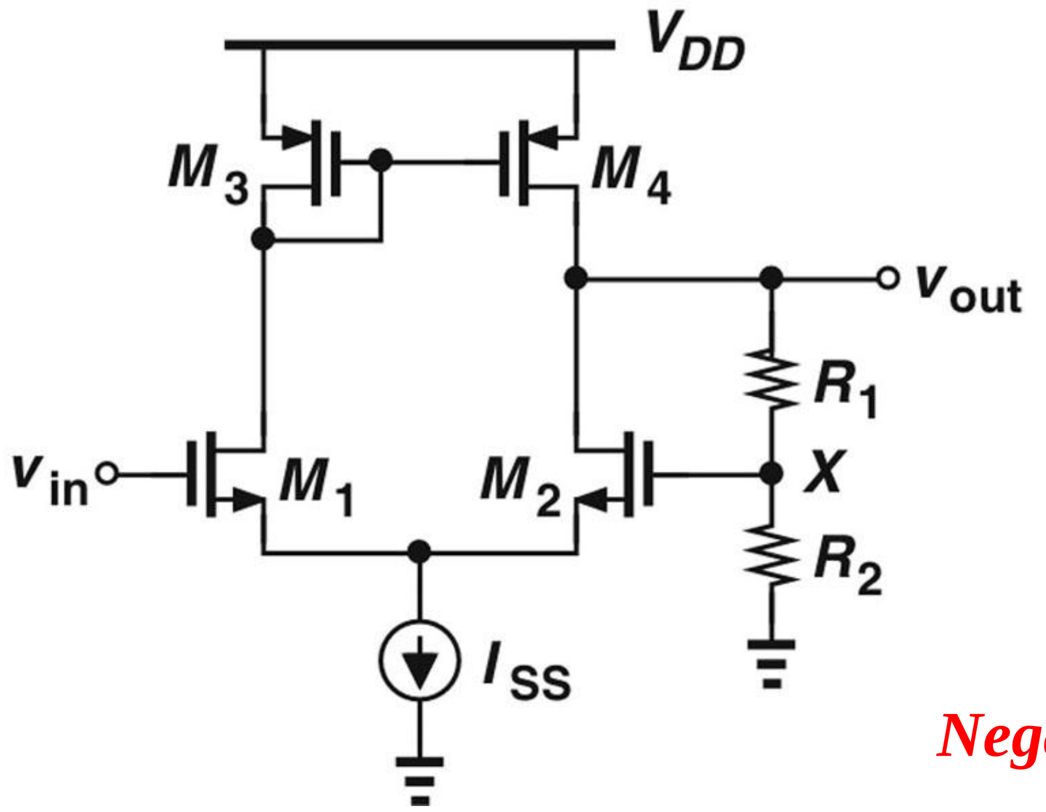
Procedure to determine the polarity:

1) Assume the input signal goes up (or down).
2) Follow the change through the forward amplifier and the feedback network.
3) Determine whether the returned quantity *enhances* or *opposes* the original change.

A simpler procedure:

1) Set the input to zero.
2) Break the loop.
3) Apply a test signal, V_{test}, travel around the loop, examine the returned signal, V_{ret}, and determine the polarity of V_{ret}/V_{test}.

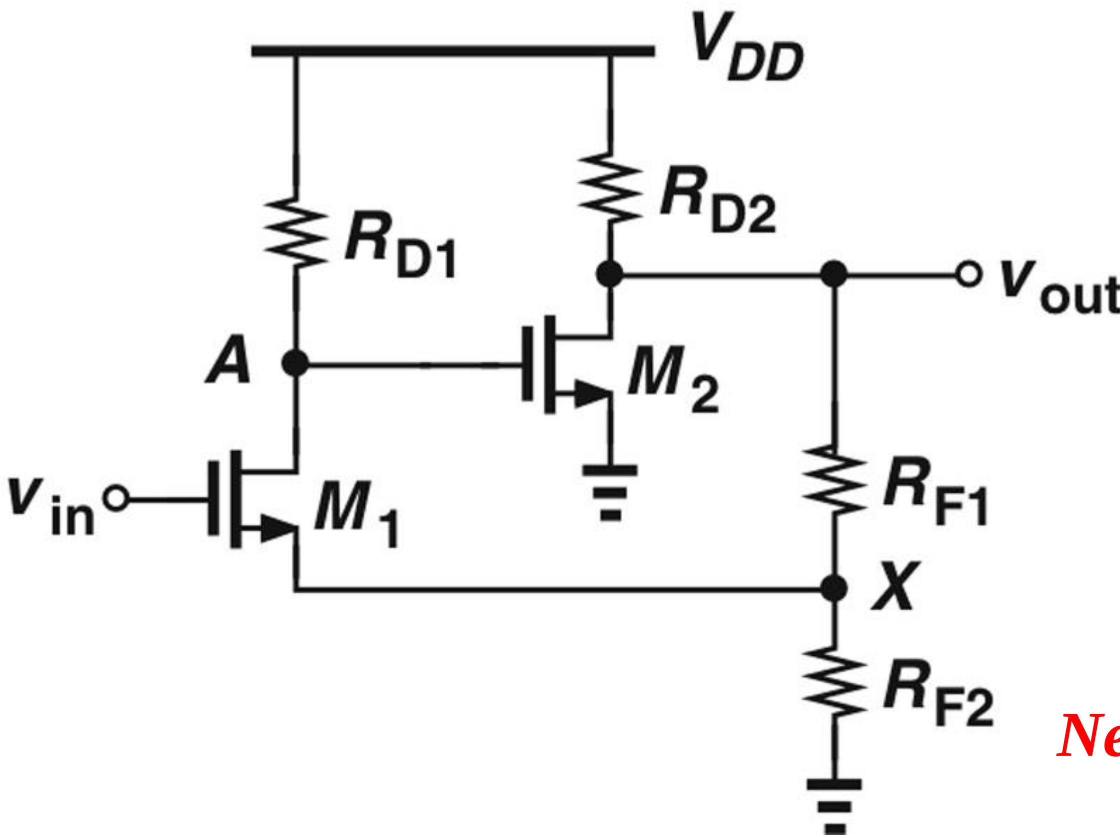
Example 1: Polarity of Feedback



Negative Feedback

$$V_{in}\uparrow \Rightarrow I_{D1}\uparrow, I_{D2}\downarrow \Rightarrow V_{out}\uparrow, V_x\uparrow \Rightarrow I_{D2}\uparrow, I_{D1}\downarrow$$

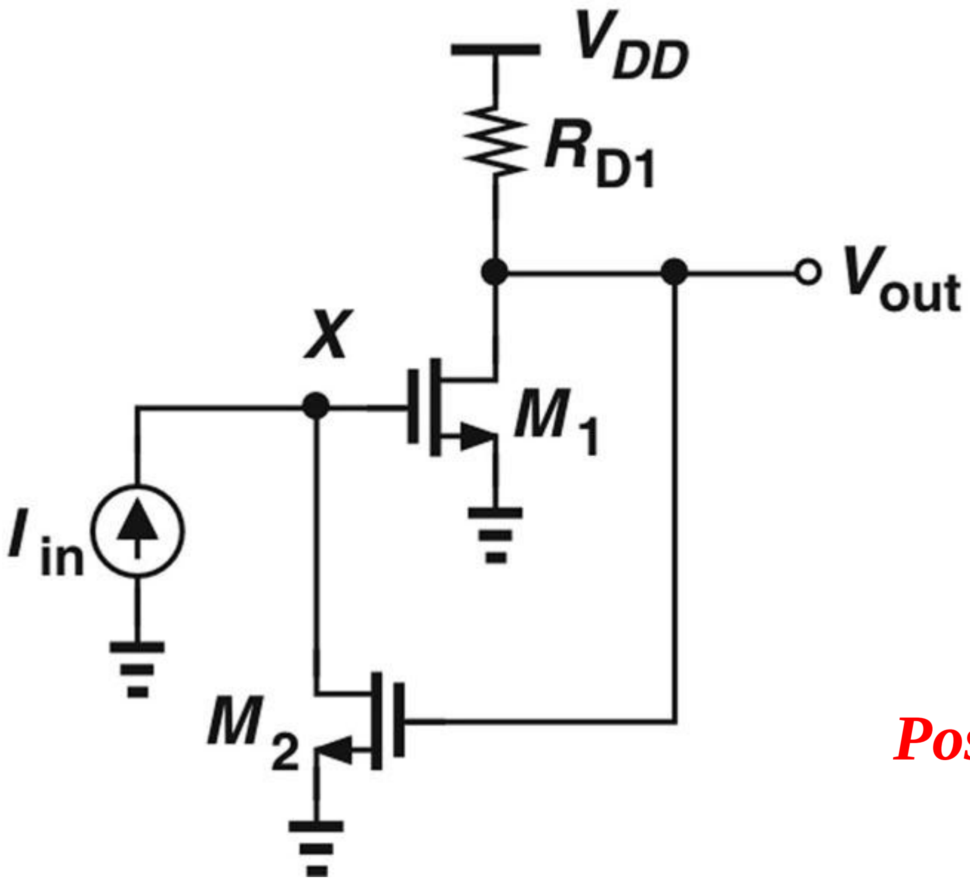
Example 2: Polarity of Feedback



Negative Feedback

$$V_{in}\uparrow \Rightarrow I_{D1}\uparrow, V_A\downarrow \Rightarrow V_{out}\uparrow, V_x\uparrow \Rightarrow I_{D1}\downarrow, V_A\uparrow$$

Example 3: Polarity of Feedback



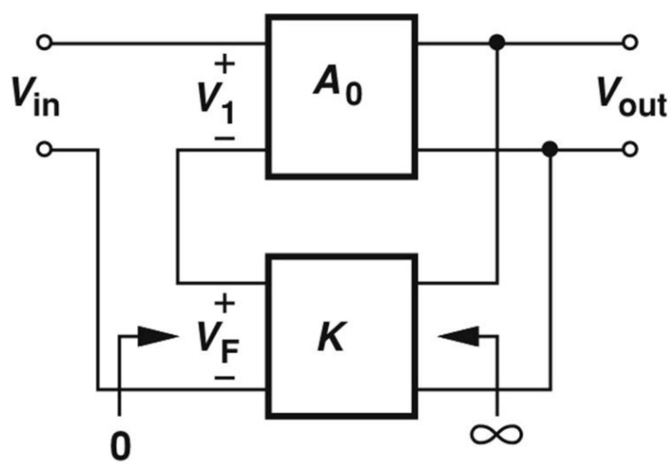
Positive Feedback

$$I_{in}\uparrow \Rightarrow I_{D1}\uparrow, V_X\uparrow \Rightarrow V_{out}\downarrow, I_{D2}\downarrow \Rightarrow I_{D1}\uparrow, V_X\uparrow$$

Feedback Topologies

- Voltage-Voltage Feedback (series-shunt)
- Voltage-Current Feedback (shunt-shunt)
- Current-Voltage Feedback (series-series)
- Current-Current Feedback (shunt-series)

Voltage-Voltage Feedback



$$V_1 = V_{in} - V_F$$

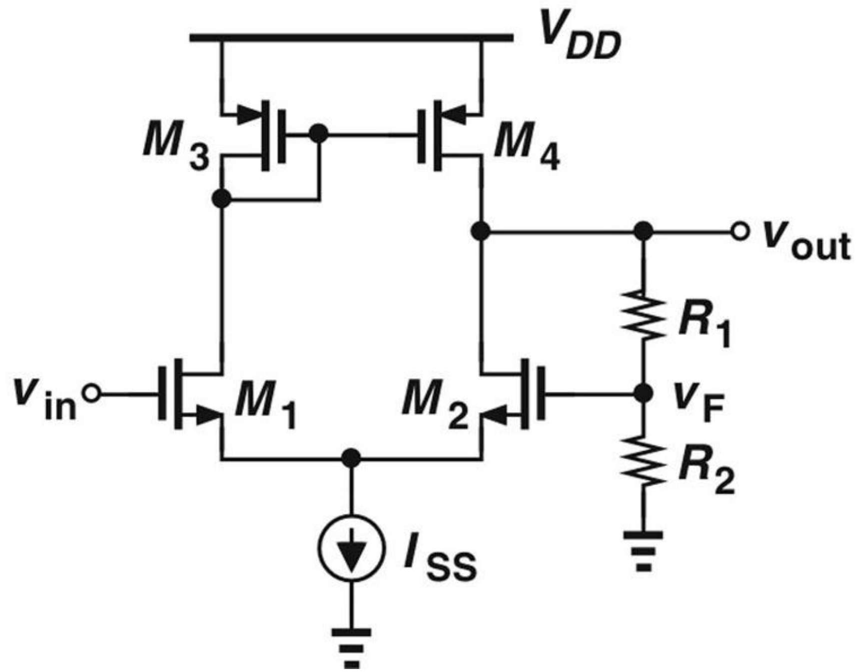
$$V_{out} = A_0 V_1$$

$$V_F = K V_{out}$$

$$V_{out} = A_0 (V_{in} - K V_{out})$$

$$\boxed{\frac{V_{out}}{V_{in}} = \frac{A_0}{1 + K A_0}}$$

Circuit realization example

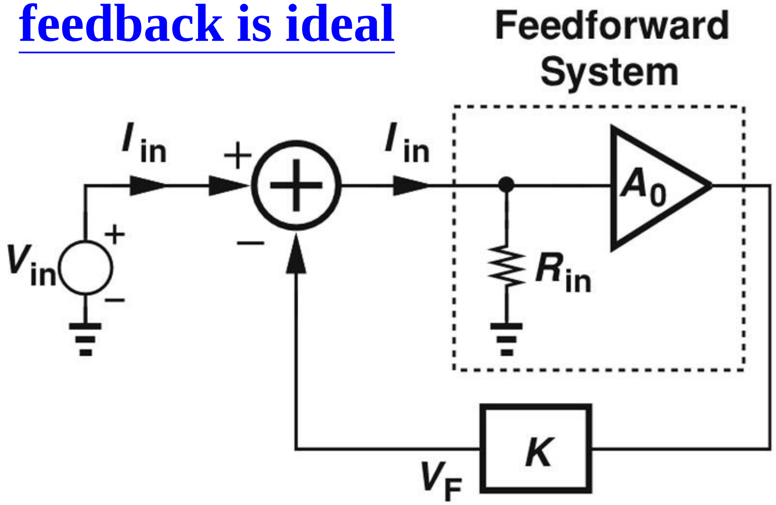


Assume $R_1 + R_2$ is very large:

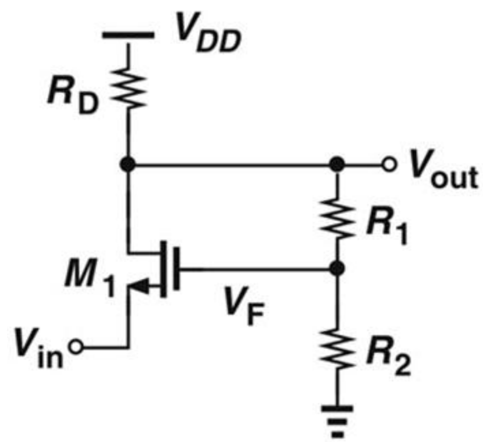
$$\frac{V_{out}}{V_{in}} = \frac{g_{mN}(r_{ON} \,||\, r_{OP})}{1 + \frac{R_2}{R_1 + R_2} g_{mN}(r_{ON} \,||\, r_{OP})}$$

Input Impedance of a V-V Feedback

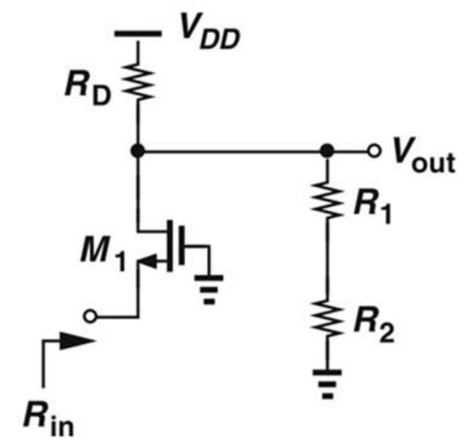
Assume forward is nonideal and feedback is ideal



Circuit example



Break the loop



$$I_{in}R_{in} = V_{in} - V_F = V_{in} - (I_{in}R_{in})A_0K$$

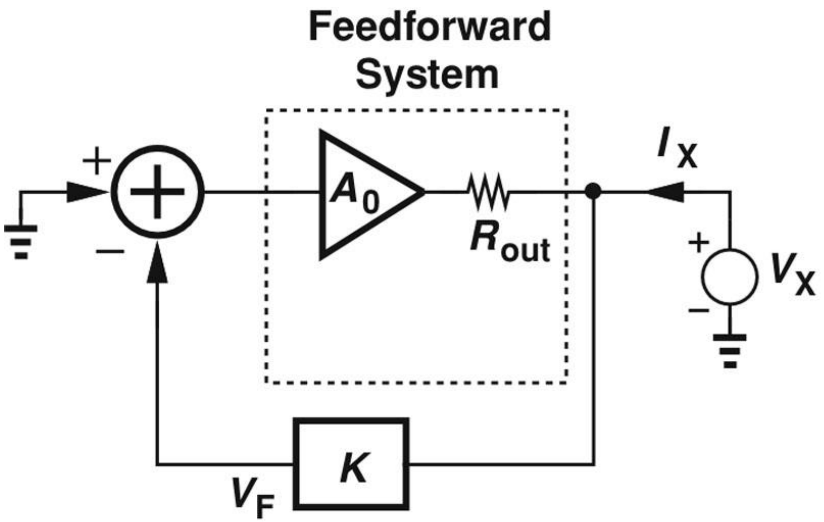
$$\Rightarrow \boxed{\frac{V_{in}}{I_{in}} = R_{in}(1+A_0K)}$$

$$R_{in} = \frac{1}{g_m}$$

$$\frac{V_{in}}{I_{in}} = \frac{1}{g_m}\left(1 + \frac{R_2}{R_1+R_2}g_mR_D\right)$$

- **The input impedance *increases* by the universal factor "1+A_0K".**
- **This impedance modification makes a more ideal voltage amplifier.**

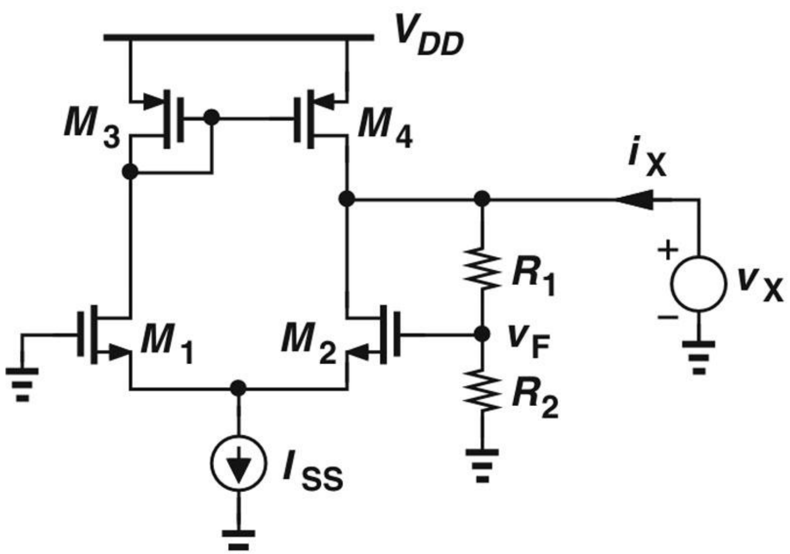
Output Impedance of a V-V Feedback



$$I_X = \frac{V_X - (-KA_0V_X)}{R_{out}}$$

$$\Rightarrow \boxed{\frac{V_X}{I_X} = \frac{R_{out}}{1+KA_0}}$$

Circuit example ($R_1 + R_2$ is very large)

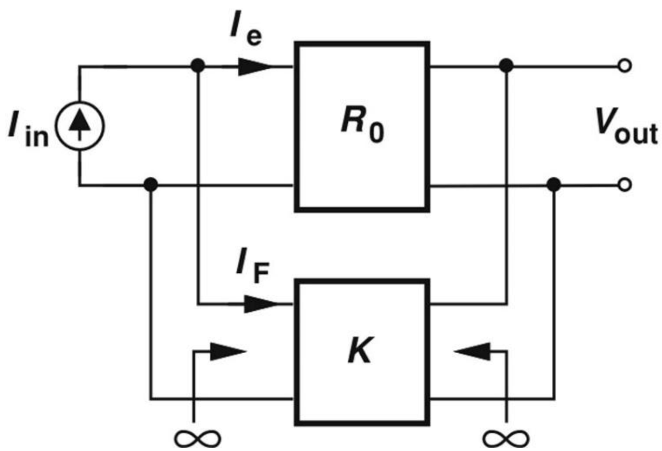


$$R_{out,closed} = \frac{r_{ON}||r_{OP}}{1 + \frac{R_2}{R_1+R_2} g_{mN}(r_{ON}||r_{OP})} \approx \left(1 + \frac{R_1}{R_2}\right)\frac{1}{g_{mN}}$$

Independent of r_{ON} and r_{OP}!

- **The output impedance *decreases* by the universal factor "$1+A_0K$".**
- **This impedance modification makes a more ideal voltage amplifier.**

Voltage-Current Feedback



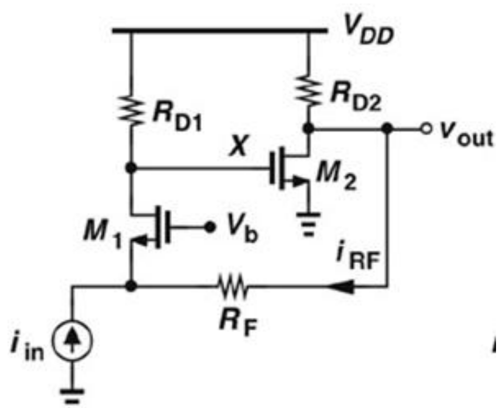
$$I_e = I_{in} - I_F$$

$$V_{out} = I_e R_0$$

$$V_{out} = (I_{in} - I_F)R_0$$
$$= (I_{in} - KV_{out})R_0$$

$$\boxed{\frac{V_{out}}{I_{in}} = \frac{R_0}{1 + KR_0}}$$

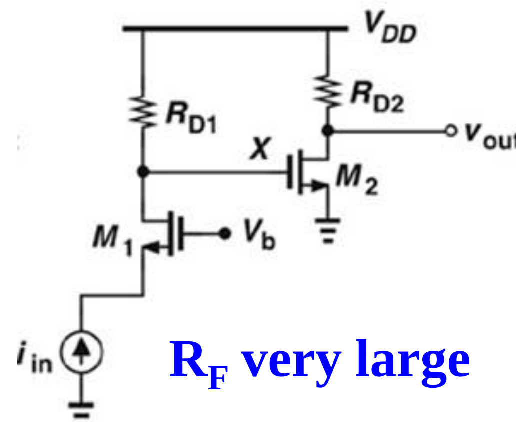
Circuit realization example (λ=0, R_F very large)



$I_{in}\uparrow$, $I_{D1}\downarrow$, $V_X\uparrow$, $V_{out}\downarrow$, $I_{RF}\downarrow$

Negative feedback

Open-loop gain



R_F very large

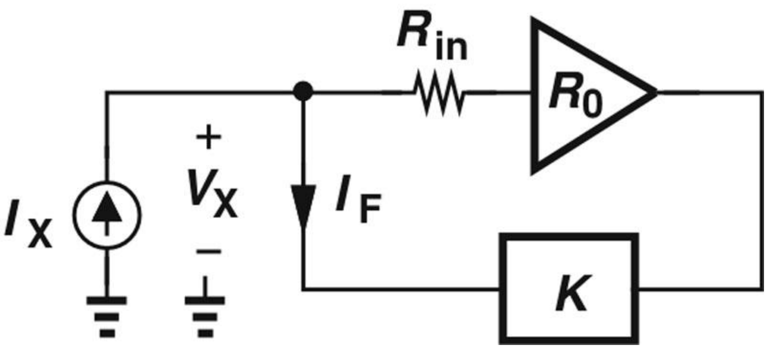
$$R_0 = R_{D1}(-g_{m2}R_{D2})$$

Closed-loop gain

$$K = -1/R_F \qquad \left.\frac{V_{out}}{I_{in}}\right|_{closed} = \frac{-g_{m2}R_{D1}R_{D2}}{1 + \dfrac{g_{m2}R_{D1}R_{D2}}{R_F}}$$

Input Impedance of a V-C Feedback

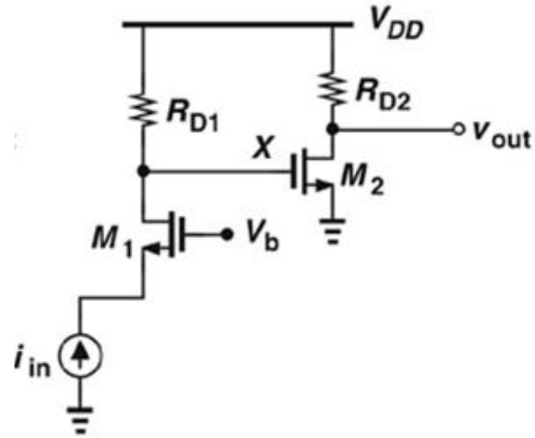
Calculation of input impedance



$$I_F = K\frac{V_X}{R_{in}}R_0 \quad I_X - K\frac{V_X}{R_{in}}R_0 = \frac{V_X}{R_{in}}$$

$$\Rightarrow \boxed{\frac{V_X}{I_X} = \frac{R_{in}}{1+KR_0}}$$

Circuit example

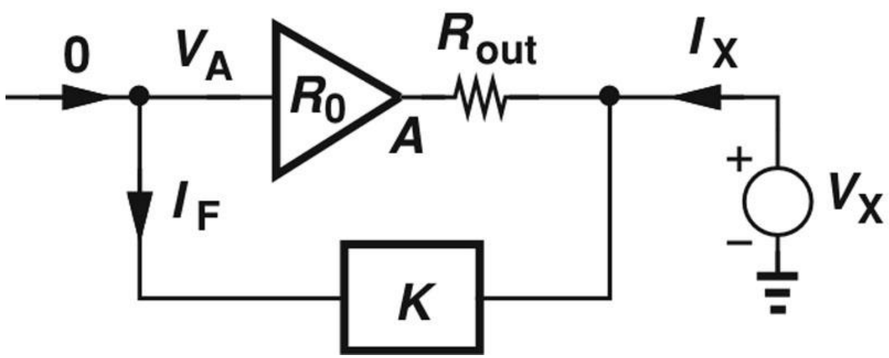


$$R_{in} = 1/g_{m1}$$

$$R_{in,closed} = \frac{1}{g_{m1}} \cdot \frac{1}{1+\frac{g_{m2}R_{D1}R_{D2}}{R_F}}$$

- **The input impedance *decreases* by the universal factor "1+R_0K".**
- **This impedance modification makes a more ideal transimpedance amplifier.**

Output Impedance of a V-C Feedback

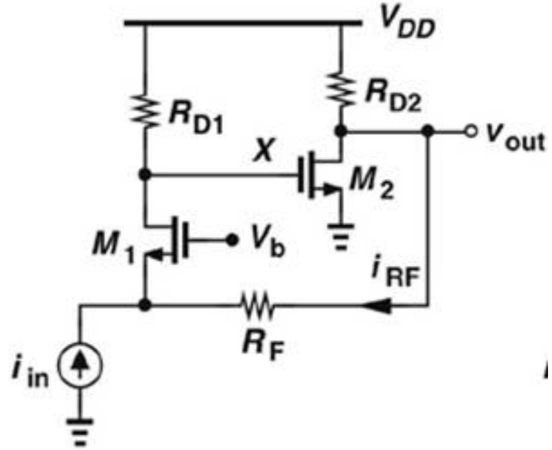


$$V_A = -KV_X R_0$$

$$I_X = \frac{V_X - V_A}{R_{out}} = \frac{V_X + KV_X R_0}{R_{out}}$$

$$\Rightarrow \boxed{\frac{V_X}{I_X} = \frac{R_{out}}{1 + KR_0}}$$

Circuit example (R_F very large)

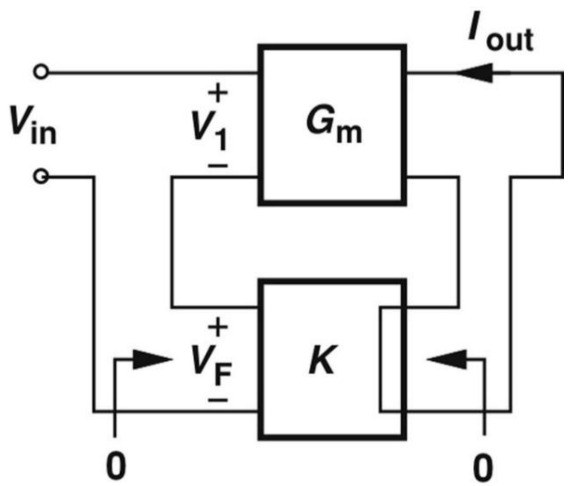


$$R_{out} \approx R_{D2}$$

$$R_{out,closed} = \frac{R_{D2}}{1 + \frac{g_{m2} R_{D1} R_{D2}}{R_F}}$$

- **The output impedance *decreases* by the universal factor "1+R_0K".**
- **This impedance modification makes a more ideal transimpedance amplifier.**

Current-Voltage Feedback



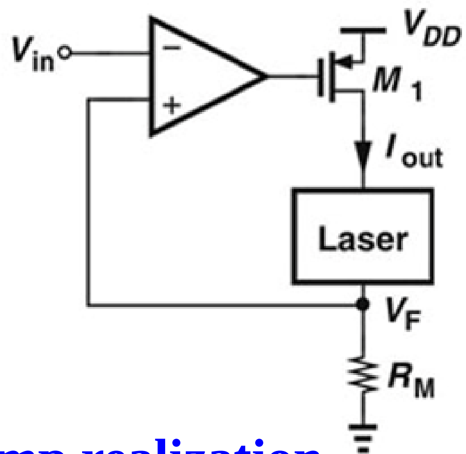
$$I_{out} = G_m(V_{in} - V_F)$$

$$V_F = KI_{out}$$

$$I_{out} = G_m(V_{in} - KI_{out})$$

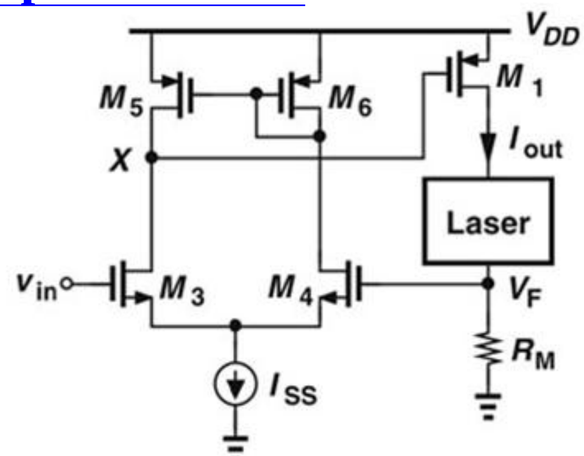
$$\boxed{\frac{I_{out}}{V_{in}} = \frac{G_m}{1 + KG_m}}$$

Circuit realization example



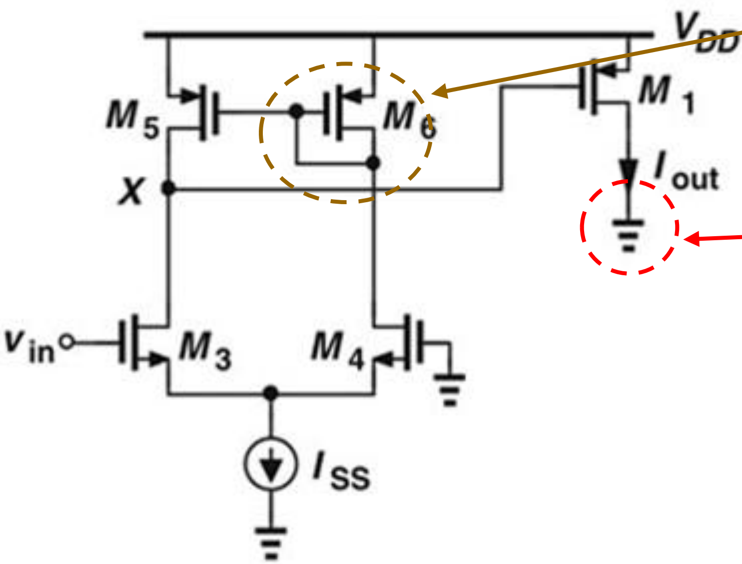
$$I_{out} \approx \frac{V_{in}}{R_M}$$

Op amp realization



Realization Example

Open-loop gain



To ensure negative feedback.
$V_{in}\uparrow$, $V_X\downarrow$, $I_{D1}\uparrow$, $V_{RM}\uparrow$

When driving no load, the output port of a circuit delivering a current must be shorted to ground.

$$K = V_F/I_{out} = R_M$$

$$I_{out} = -g_{m1}V_X$$

$$V_X = -g_{m3}(r_{O3}||r_{O5})V_{in}$$

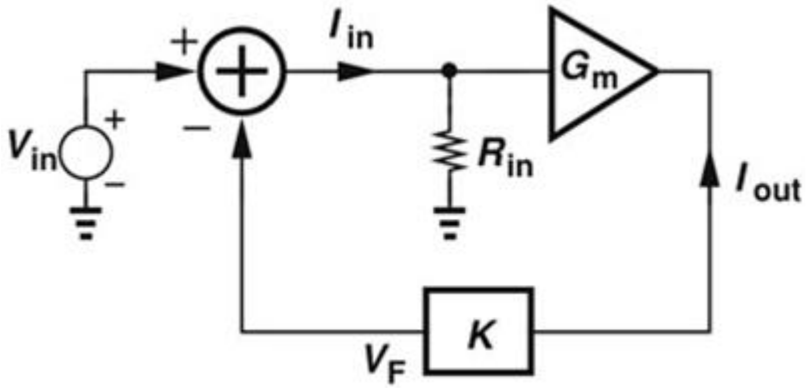
$$\Rightarrow G_m = g_{m1}g_{m3}(r_{O3}||r_{O5})$$

$$\Rightarrow \left.\frac{I_{out}}{V_{in}}\right|_{closed} = \frac{g_{m1}g_{m3}(r_{O3}||r_{O5})}{1+g_{m1}g_{m3}(r_{O3}||r_{O5})R_M}$$

$$\Rightarrow \left.\frac{I_{out}}{V_{in}}\right|_{closed} \approx \frac{1}{R_M}$$

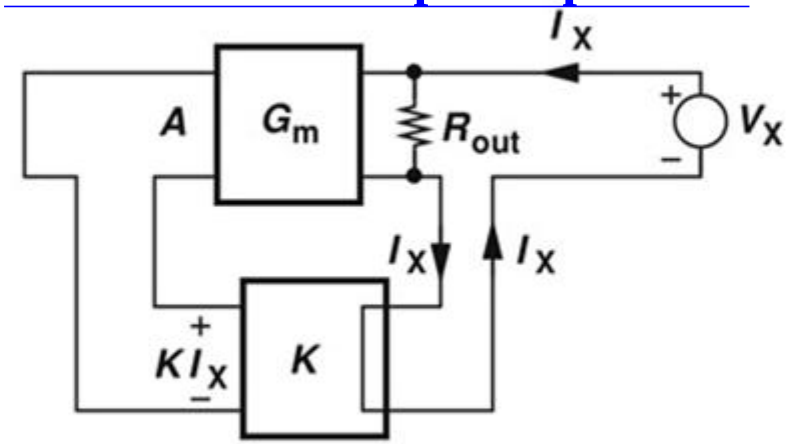
I/O Impedances of a C-V Feedback

Calculation of input impedance



$$\frac{V_{in}}{I_{in}} = R_{in}(1 + KG_m)$$

Calculation of output impedance

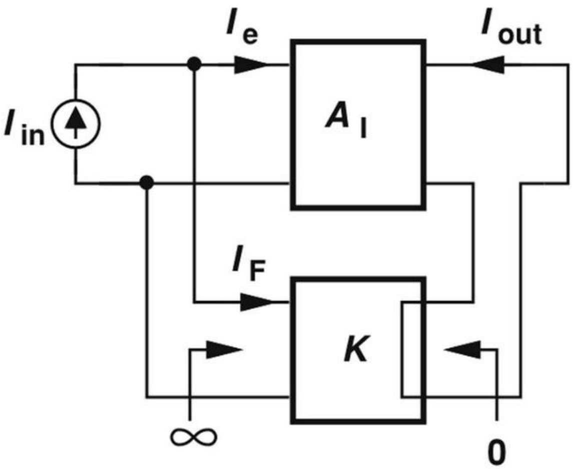


$$I_X = \frac{V_X}{R_{out}} - KG_m I_X$$

$$\frac{V_X}{I_X} = R_{out}(1 + KG_m)$$

- **The I/O impedances *increase* by the universal factor "1+G_mK".**
- **These impedance modifications make a more ideal transconductance amplifier.**

Current-Current Feedback

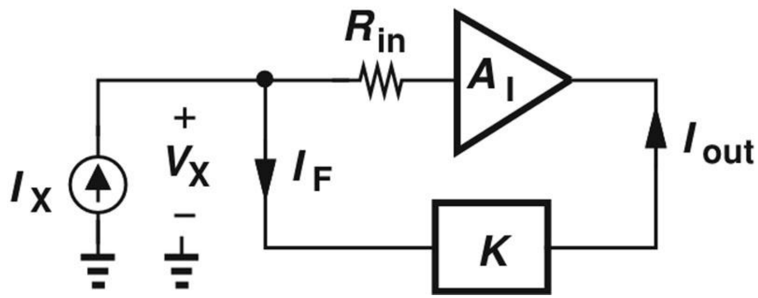


$$I_{out} = A_I(I_{in} - I_F)$$

$$= A_I(I_{in} - KI_{out})$$

$$\boxed{\frac{I_{out}}{I_{in}} = \frac{A_I}{1 + KA_I}}$$

Input impedance

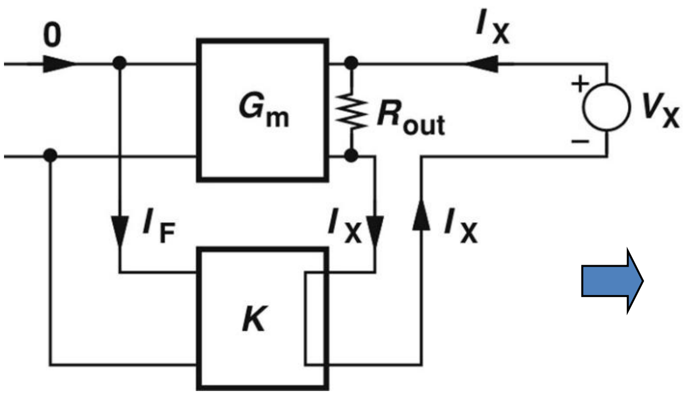


$$I_X = \frac{V_X}{R_{in}} + I_F$$

$$= \frac{V_X}{R_{in}} + KA_I \frac{V_X}{R_{in}}$$

$$\boxed{\frac{V_X}{I_X} = \frac{R_{in}}{1 + KA_I}}$$

Output impedance

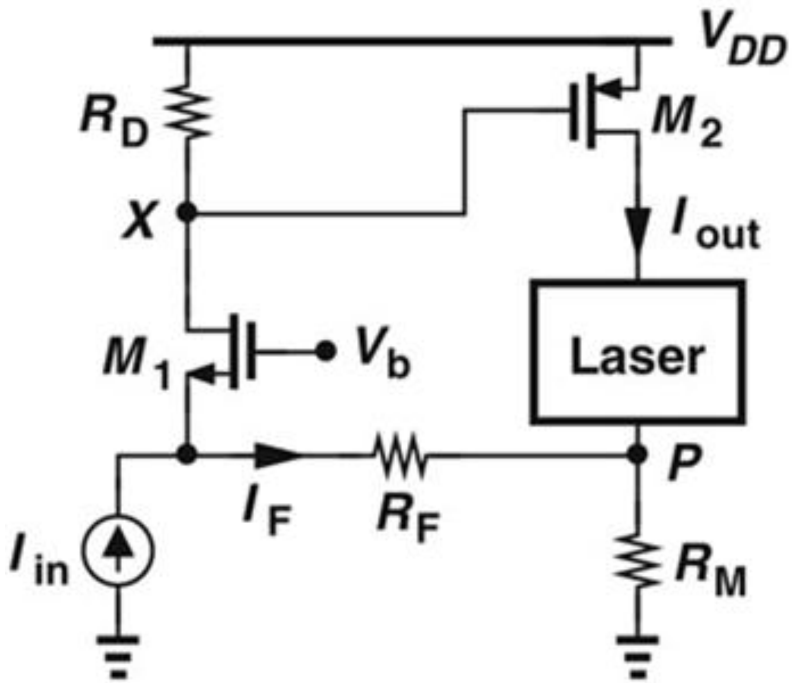


$$I_X = \frac{V_X}{R_{out}} - KA_I I_X$$

$$\boxed{\frac{V_X}{I_X} = R_{out}(1 + KA_I)}$$

- **The I/O impedances *de/increase* by the universal factor “1+A_IK”.**
- **These impedance modifications make a more ideal current amplifier.**

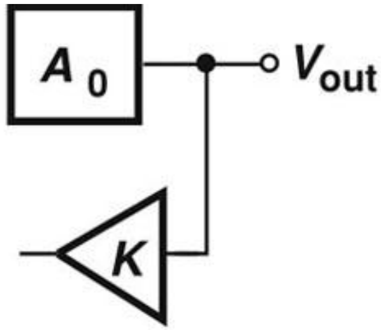
Example 4: Polarity of Feedback



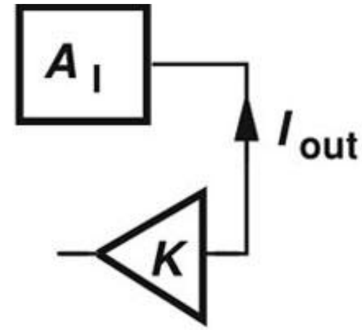
Negative Feedback

$$I_{in}\uparrow \Rightarrow V_{D1}\uparrow, I_{out}\downarrow \Rightarrow V_P\downarrow, I_F\uparrow \Rightarrow V_{D1}\downarrow, I_{out}\uparrow$$

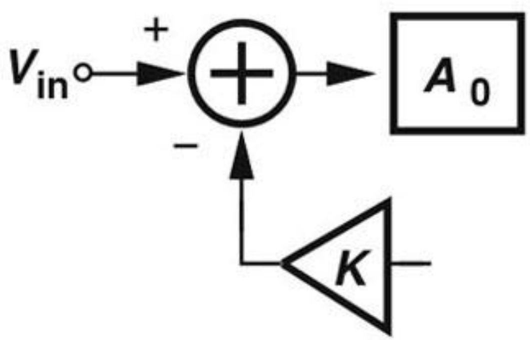
Effect of Feedback on I/O Impedances



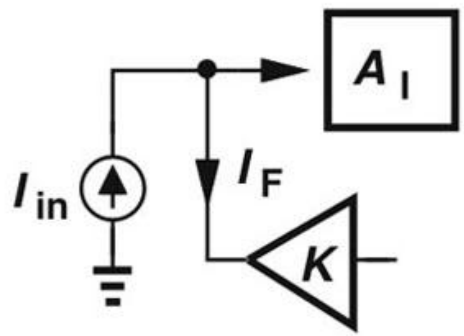
Output impedance falls by 1+ loop gain.



Output impedance rises by 1+ loop gain.



Input impedance rises by 1+ loop gain.



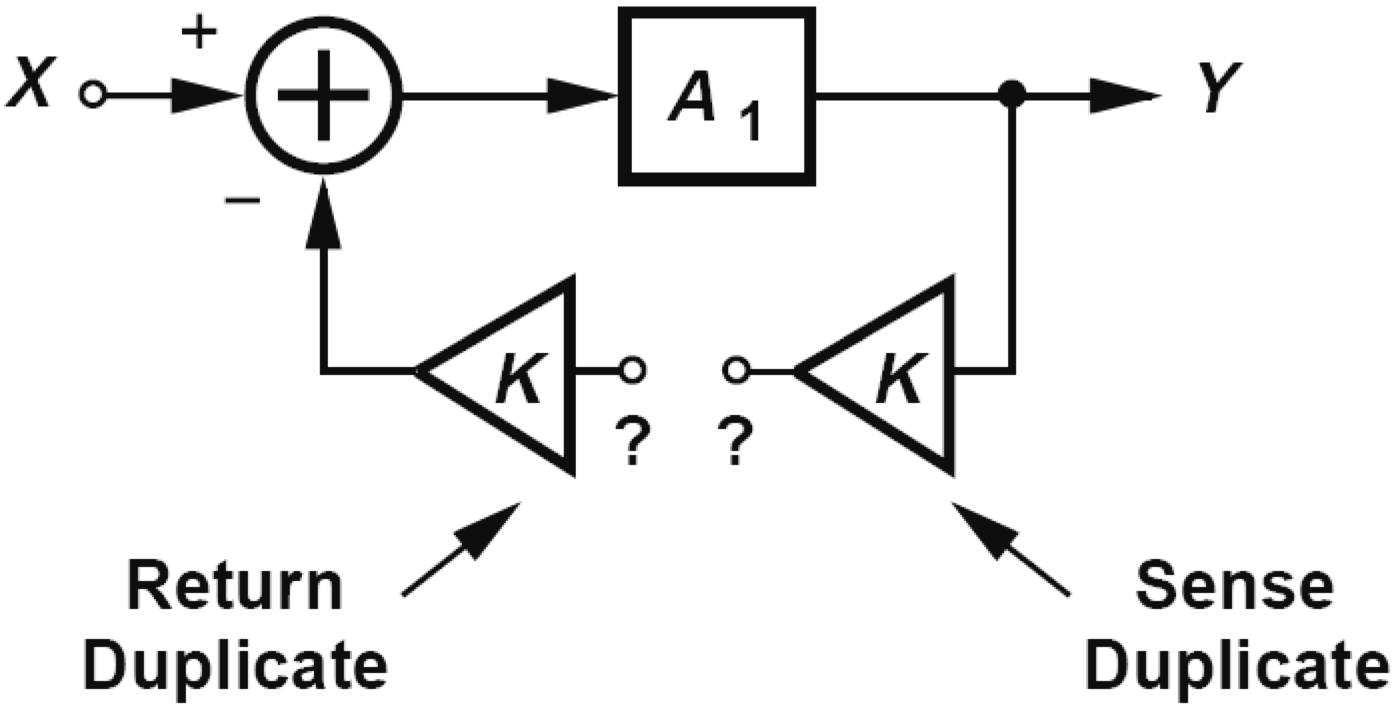
Input impedance falls by 1+ loop gain.

Inclusion of I/O Effects

A methodology that allows the analysis even if the I/O impedances of the forward amplifier or the feedback network depart from their ideal values.

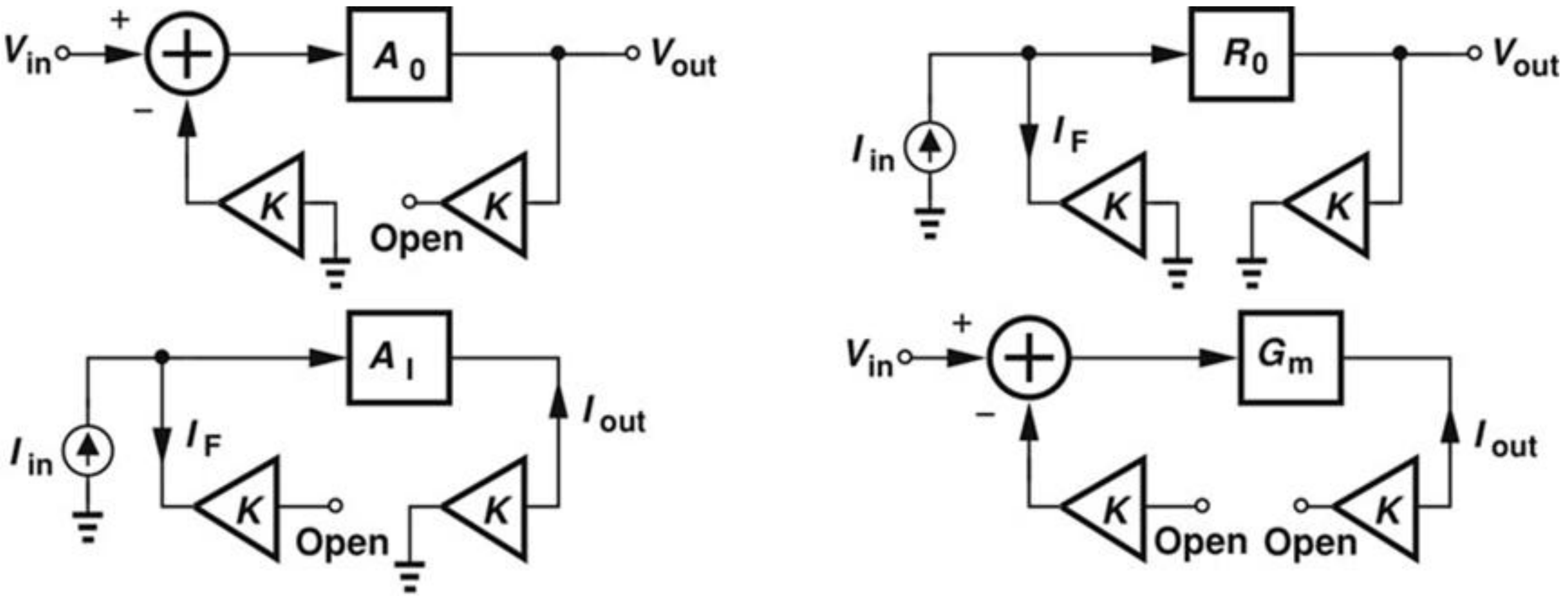
1) Identify the forward amplifier.
2) Identify the feedback network.
3) Break the feedback network according to the rules described below.
4) Calculate the open-loop parameters.
5) Determine the feedback factor according to the rules described below.
6) Calculate the closed-loop parameters.

Step 3: How to Break a Loop?



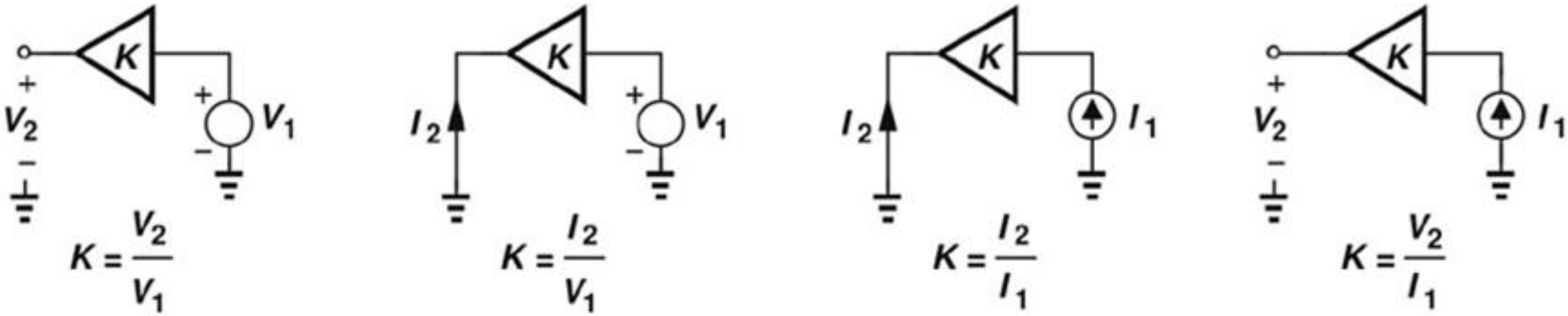
- **The correct way of breaking a loop is such that the loop does not know it has been broken. Therefore, we need to present the feedback network to both the input and the output of the feedforward amplifier.**

Termination Rules of Breaking a Loop



- Since ideally, the input of the feedback network sees zero impedance (Z_{out} of an ideal voltage source), the return replicate needs to be grounded. Similarly, the output of the feedback network sees an infinite impedance (Z_{in} of an ideal voltage sensor), the sense replicate needs to be open.
- Similar ideas apply to the other types.

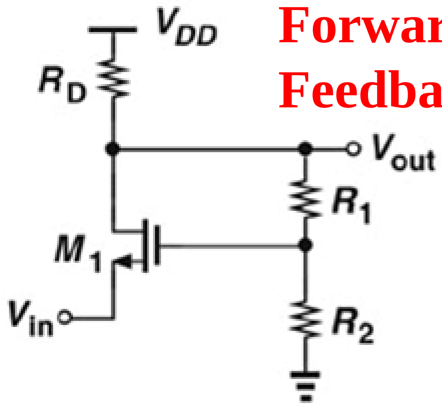
Step 5: Calculation of Feedback Factor



- Since the feedback senses voltage, the input of the feedback is a voltage source. Moreover, since the return quantity is also voltage, the output of the feedback is left open (a short means the output is always zero).
- Similar ideas apply to the other types.

Example 12.26

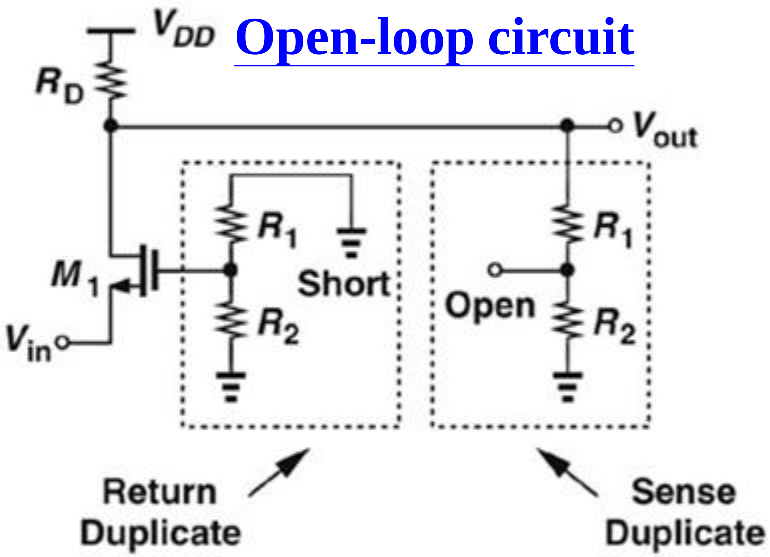
$R_1 + R_2$ is not much greater than R_D



Forward: M_1, R_D

Feedback: R_1, R_2

Open-loop circuit

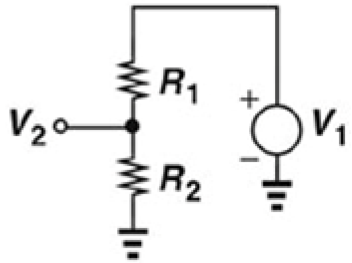


Open-loop parameters

$$A_O = g_{m1}[R_D||(R_1+R_2)]$$

$$R_{out,open} = R_D||(R_1+R_2)$$

Feedback factor



$$K = \frac{V_2}{V_1} = \frac{R_2}{R_1+R_2}$$

$$KA_0 = \frac{R_2}{R_1+R_2} g_{m1}[R_D||(R_1+R_2)]$$

$$A_{v,closed} = \frac{g_{m1}[R_D||(R_1+R_2)]}{1+\frac{R_2}{R_1+R_2} g_{m1}[R_D||(R_1+R_2)]}$$

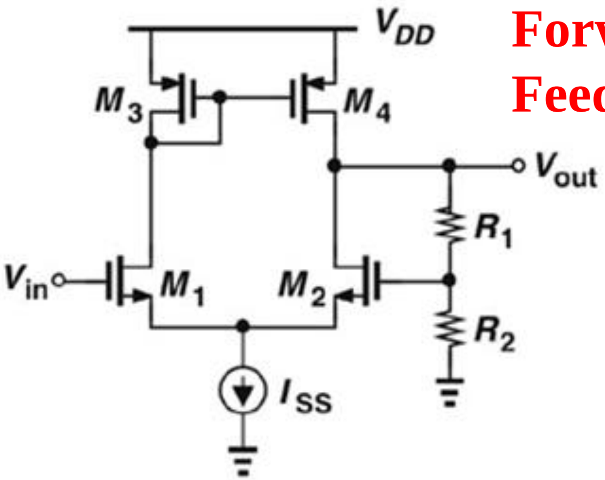
$$R_{in,closed} = \frac{1}{g_{m1}}\left\{1+\frac{R_2}{R_1+R_2} g_{m1}[R_D||(R_1+R_2)]\right\}$$

$$R_{out,closed} = \frac{R_D||(R_1+R_2)}{1+\frac{R_2}{R_1+R_2} g_{m1}[R_D||(R_1+R_2)]}$$

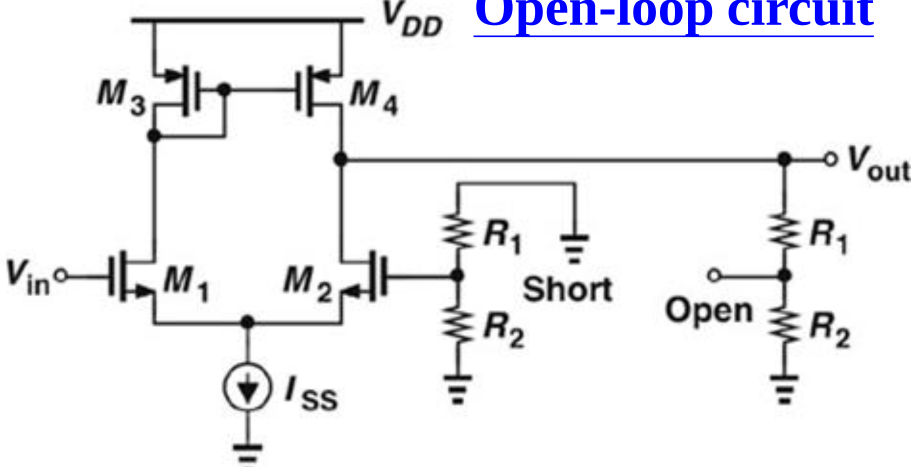
Example 12.27

$R_1 + R_2$ is not much greater than $r_{OP}||r_{ON}$

Forward: M_1-M_4
Feedback: R_1, R_2



Open-loop circuit



Open-loop parameters

$$A_0 = g_{mN}[r_{ON}||r_{OP}||(R_1 + R_2)]$$

$$R_{in,open} = \infty$$

$$R_{out,open} = r_{ON}||r_{OP}||(R_1 + R_2).$$

Feedback factor

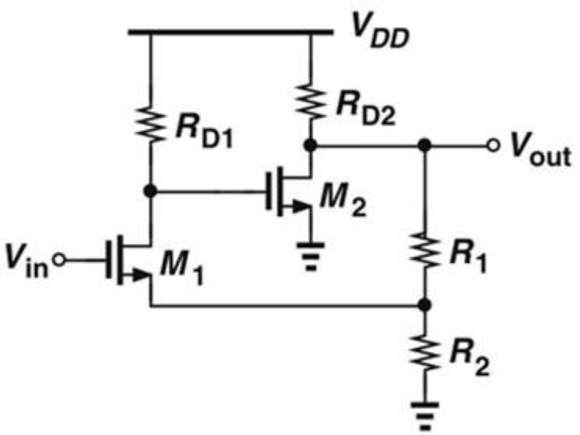
$$K = \frac{R_2}{R_1 + R_2}$$

$$\left.\frac{V_{out}}{V_{in}}\right|_{closed} = \frac{g_{mN}[r_{ON}||r_{OP}||(R_1 + R_2)]}{1 + \dfrac{R_2}{R_1 + R_2} g_{mN}[r_{ON}||r_{OP}||(R_1 + R_2)]}$$

$$R_{in,closed} = \infty$$

$$R_{out,closed} = \frac{r_{ON}||r_{OP}||(R_1 + R_2)}{1 + \dfrac{R_2}{R_1 + R_2} g_{mN}[r_{ON}||r_{OP}||(R_1 + R_2)]}$$

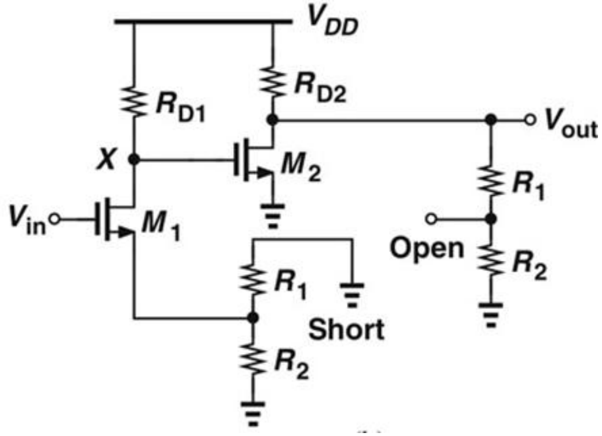
Example 12.28



Forward: M_1, R_{D1}, M_2, R_{D2}

Feedback: R_1, R_2 to M_1

Open-loop circuit



Open-loop parameters

$$A_0 = \frac{-R_{D1}}{\dfrac{1}{g_m} + R_1||R_2} \cdot \{-g_{m2}[R_{D2}||(R_1+R_2)]\}$$

$$R_{in,open} = \infty$$

$$R_{out,open} = R_{D2}||(R_1+R_2)$$

Feedback factor

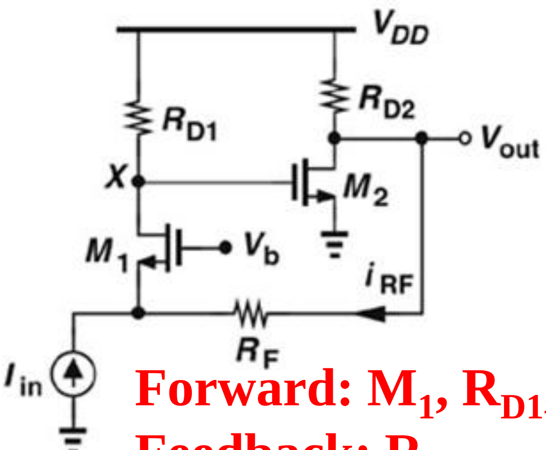
$$K = \frac{R_2}{R_1+R_2}$$

$$\left.\frac{V_{out}}{V_{in}}\right|_{closed} = \frac{A_0}{1+\dfrac{R_2}{R_1+R_2}A_0}$$

$$R_{in,closed} = \infty$$

$$R_{out,closed} = \frac{R_{D2}||(R_1+R_2)}{1+\dfrac{R_2}{R_1+R_2}A_0}$$

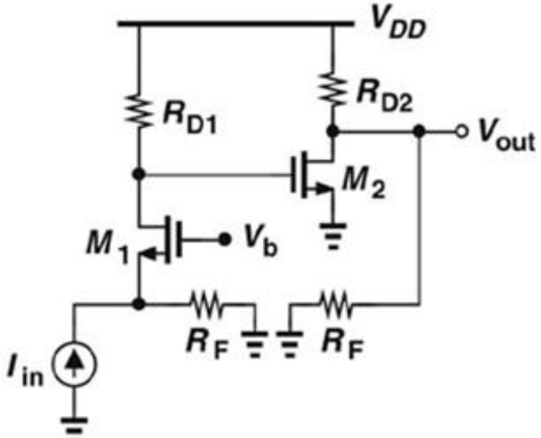
R_F is not very large



Forward: M_1, R_{D1}, M_2, R_{D2}

Feedback: R_F

Open-loop circuit



Open-loop parameters

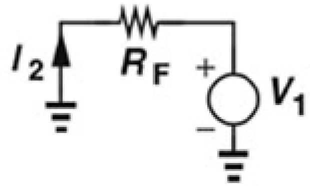
$$V_X = I_{in} \frac{R_F R_{D1}}{R_F + \frac{1}{g_{m1}}}$$

$$R_0 = V_{out}/I_{in} = (V_X/I_{in})(V_{out}/V_X)$$

$$\Rightarrow R_0 = \frac{R_F R_{D1}}{R_F + \frac{1}{g_{m1}}} \cdot [-g_{m2}(R_{D2}||R_F)]$$

$$R_{in,open} = \frac{1}{g_{m1}}||R_F \qquad R_{out,open} = R_{D2}||R_F$$

Feedback factor

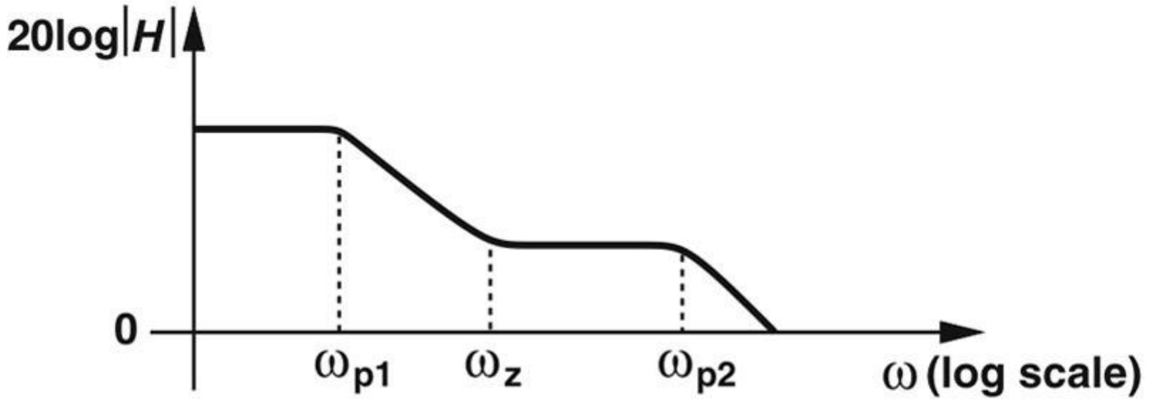


$$K = \frac{I_2}{V_1} = -\frac{1}{R_F}$$

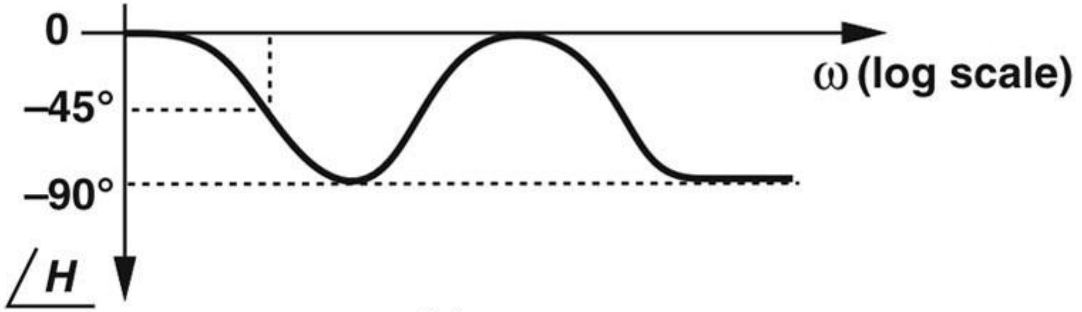
$$\left.\frac{V_{out}}{I_{in}}\right|_{closed} = \frac{R_0}{1 - \frac{R_0}{R_F}} \qquad R_{in,closed} = \frac{\frac{1}{g_{m1}}||R_F}{1 - \frac{R_0}{R_F}} \qquad R_{out,closed} = \frac{R_{D2}||R_F}{1 - \frac{R_0}{R_F}}$$

Phase Response Using Bode's Rule

Assume ω_{p1}, ω_z, ω_{p2} are far apart



(a)

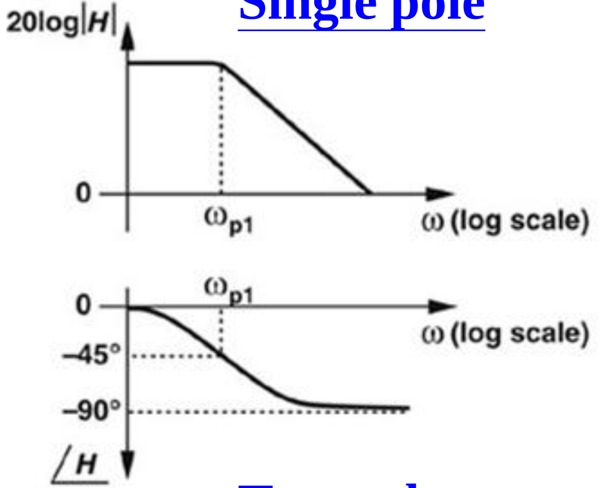


(b)

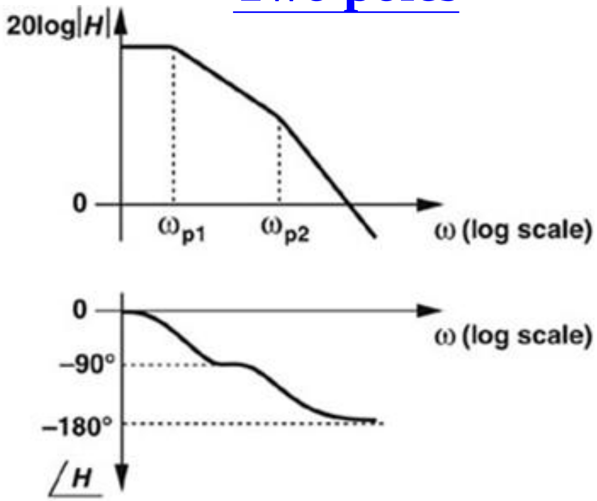
- **As it can be seen, the phase of H(jω) starts to drop at 1/10 of the pole, hits -45° at the pole, and approaches -90° at 10 times the pole.**

Amplifier with Multiple poles

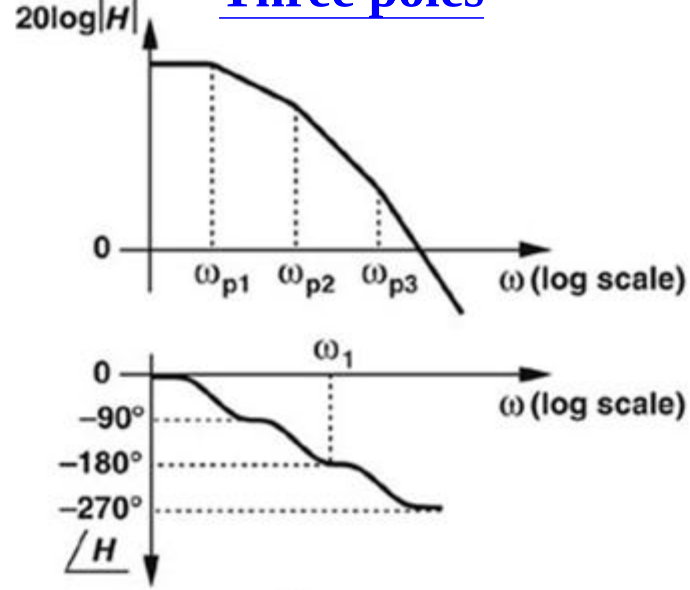
Single pole

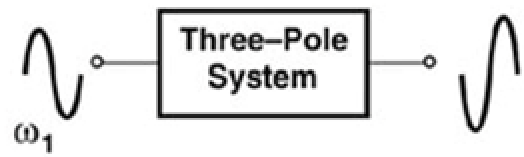


Two poles



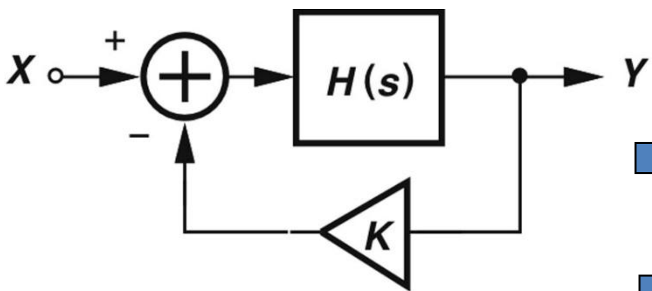
Three poles





- **The three-pole system introduces a phase shift of -180° at a finite frequency ω1, reversing the sign of an input sinusoid at this frequency.**

Problem of Instability



$$\frac{Y}{X}(s) = \frac{H(s)}{1+KH(s)}$$

KH(s): loop transmission

What happens when $KH(j\omega_1) = -1$?

The closed-loop system provides infinite gain!

Even an infinitesimally small input at ω_1 leads to a finite output component at this frequency!

The system *oscillates* at ω_1.

Intuitive understanding:

The feedback becomes *positive* at ω_1, producing a feedback signal that *enhances* the input.

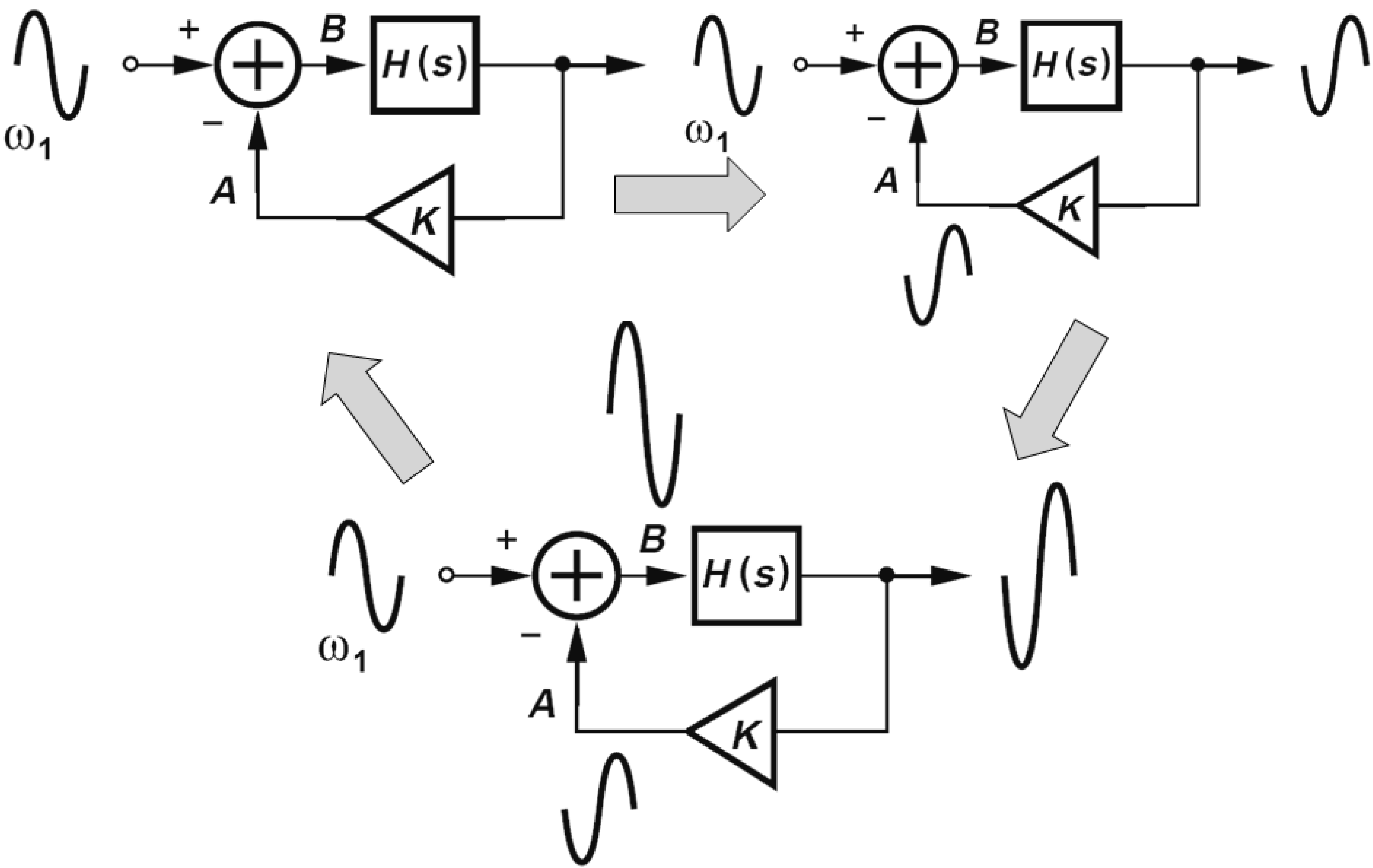
The final amplitude bounded by the supply voltage or other "saturation" mechanisms.

Barkhausen's criteria for oscillation

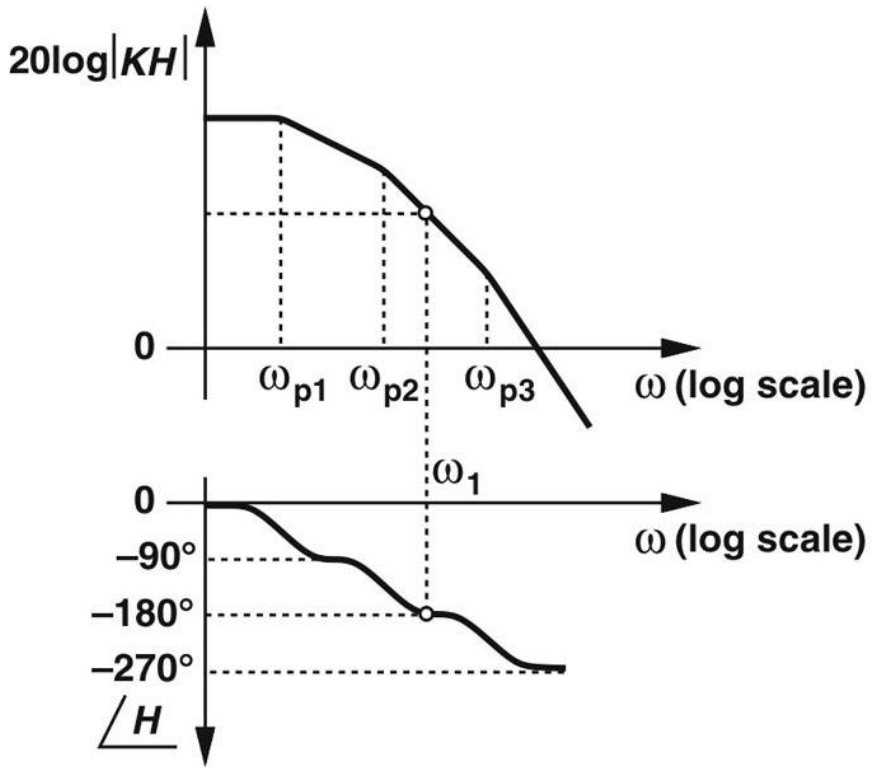
$KH(j\omega_1) = -1$ $\Rightarrow$

$$|KH(j\omega_1| = 1$$
$$\angle KH(j\omega_1) = -180°$$

Time Evolution of Instability

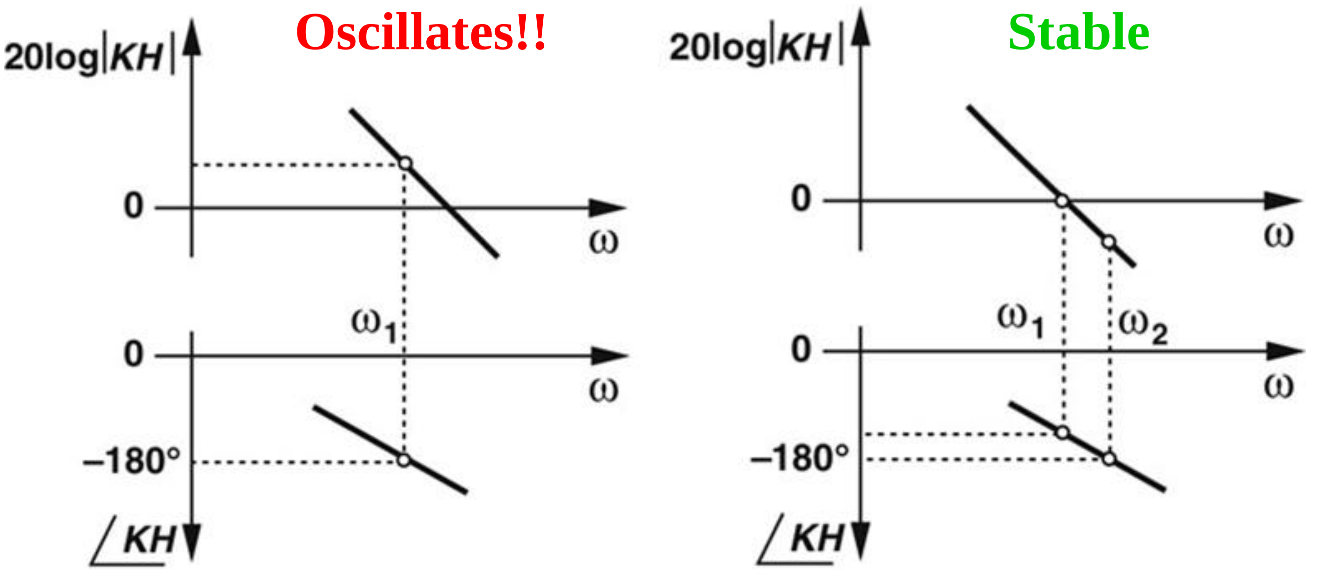


Oscillation Example



- This system oscillates, since there's a finite frequency at which the phase is -180° and the gain is greater than unity. In fact, this system exceeds the minimum oscillation requirement.

Stability Condition



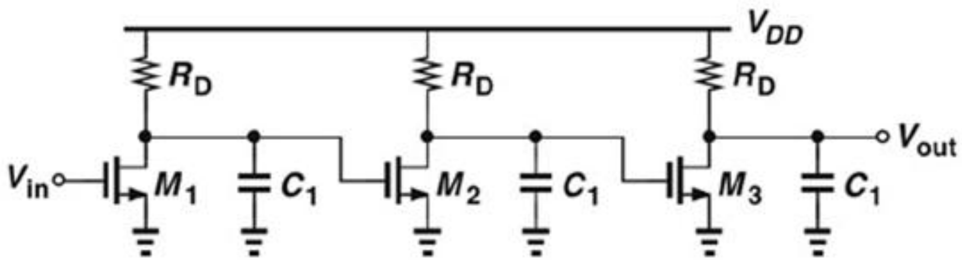
Stability

$$\omega_{GX} < \omega_{PX}$$

ω_{PX}, (“phase crossover”), is the frequency at which ∠KH=-180°.
ω_{GX}, (“gain crossover”), is the frequency at which |KH|=1.

- **To guarantee stability in negative-feedback systems, we must ensure that the loop gain falls to unity before the phase shift reaches -180° so that Barkhausen’s criteria do not hold at the same frequency.**

Stability Example

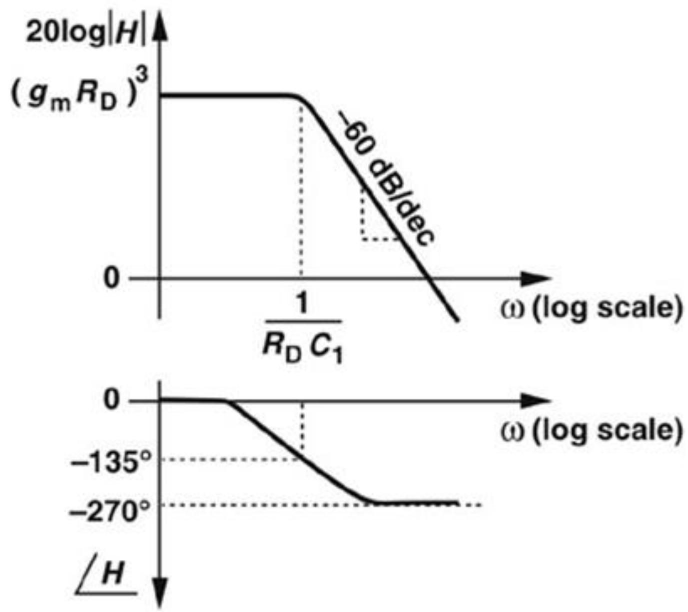


Apply negative feedback with K=1

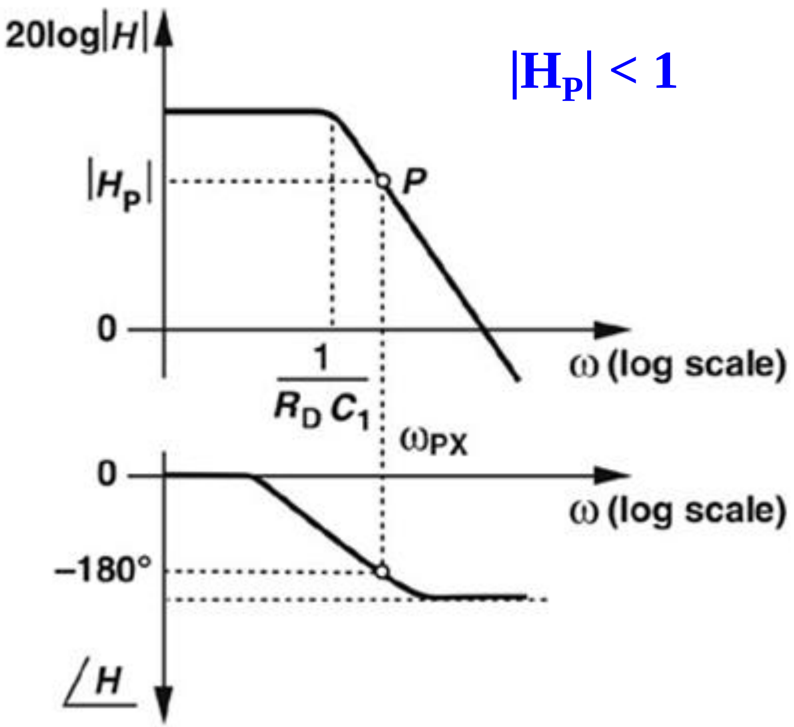
Ensure |KH| falls below unity at ω_{PX}

Low frequency gain: $(g_m R_D)^3$

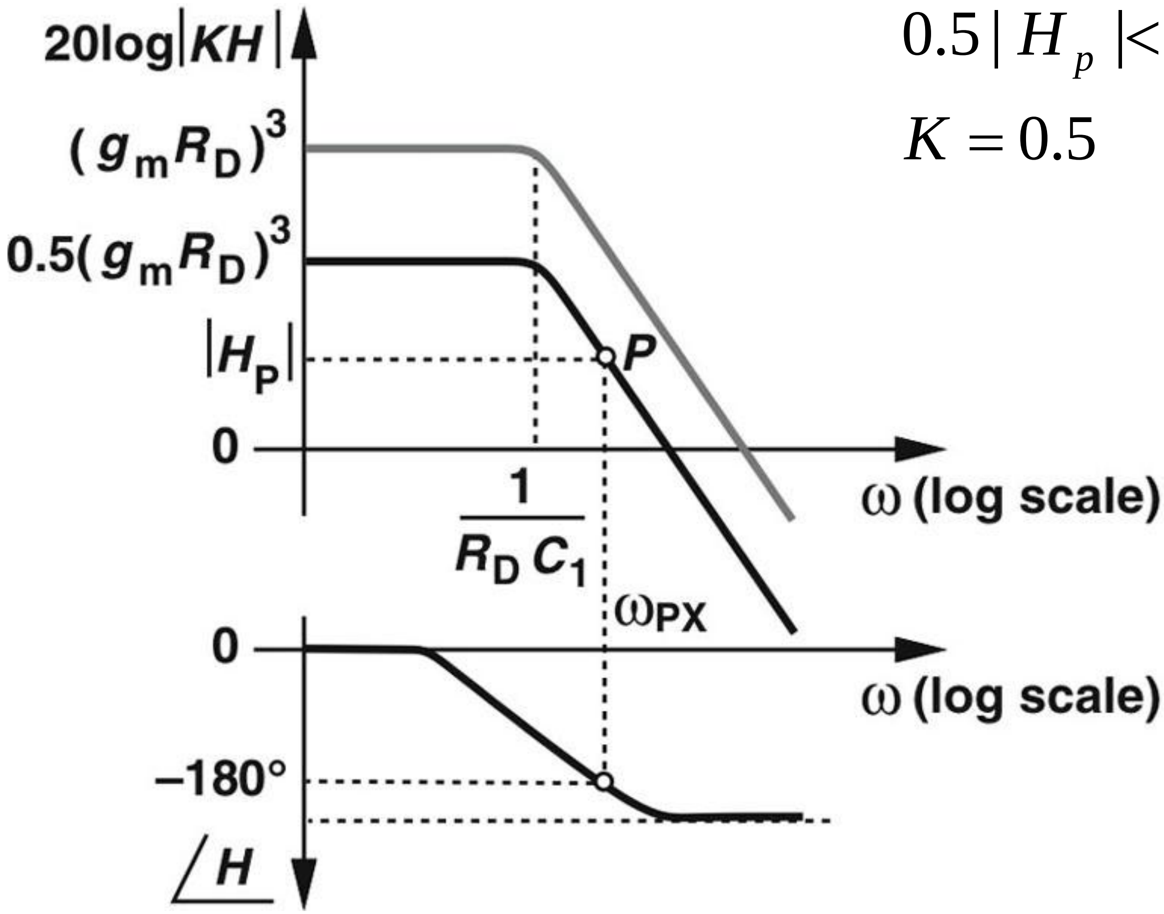
Three coincident poles at $(R_D C_1)^{-1}$



$|H_P| < 1$



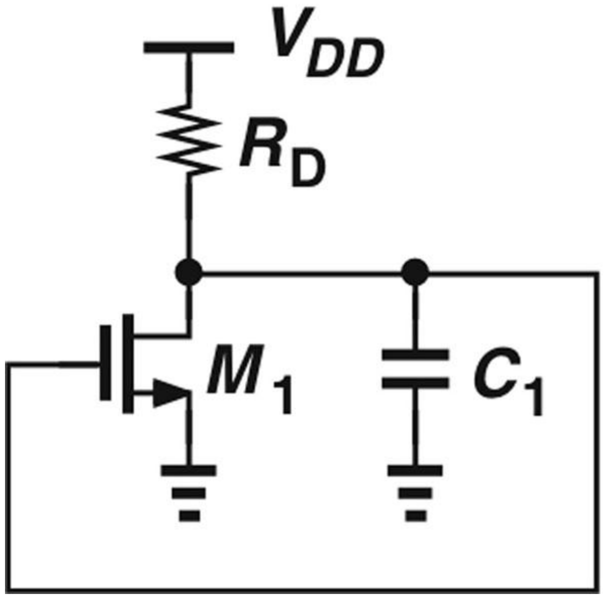
If the Feedback is Weaker (K=0.5)



$$0.5\,|H_p| < 1$$

$$K = 0.5$$

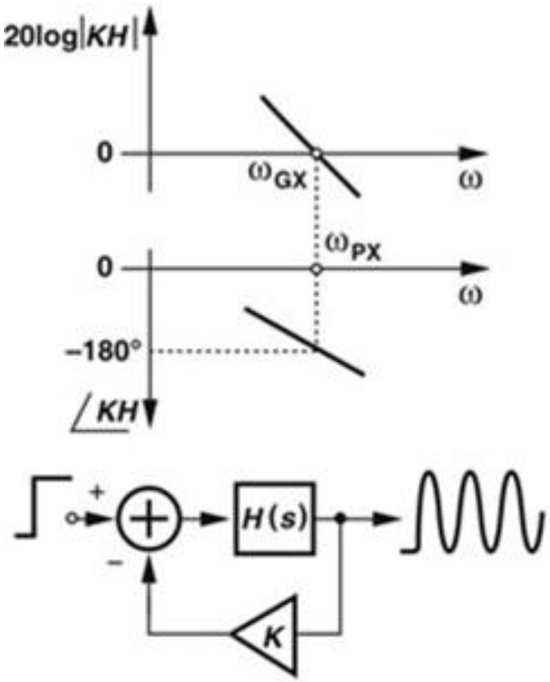
Will this Circuit Oscillate?



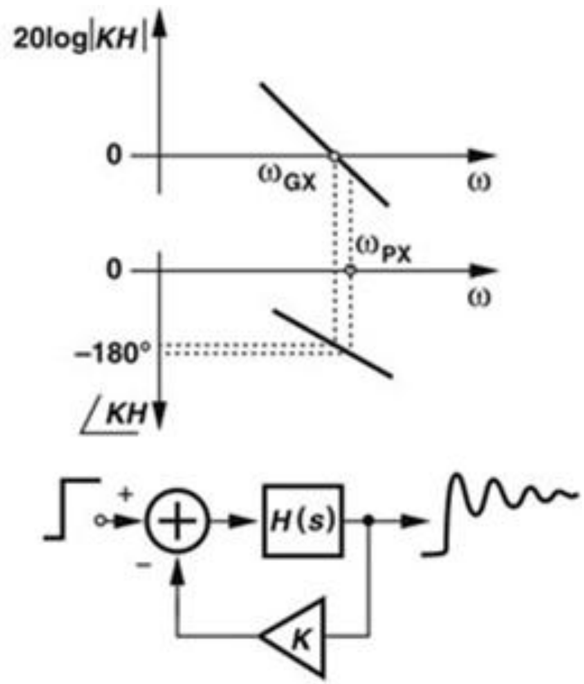
- Since the circuit contains only one pole, the phase shift cannot reach 180° at any frequency. The circuit is thus stable.

Different Stability Conditions

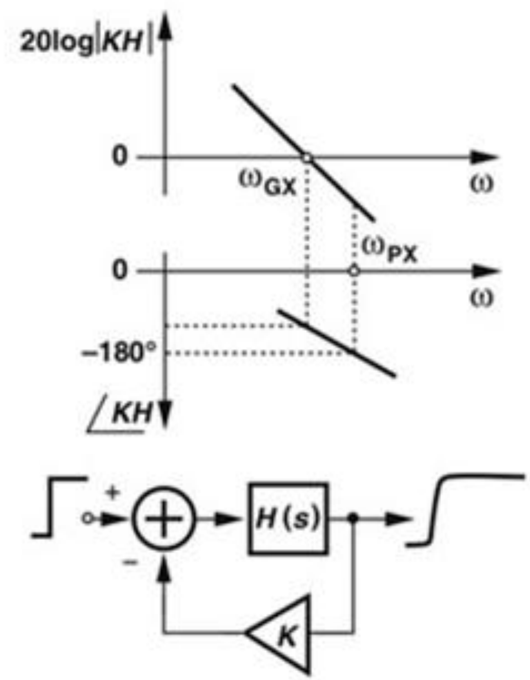
Oscillation



Marginally stable



Stable

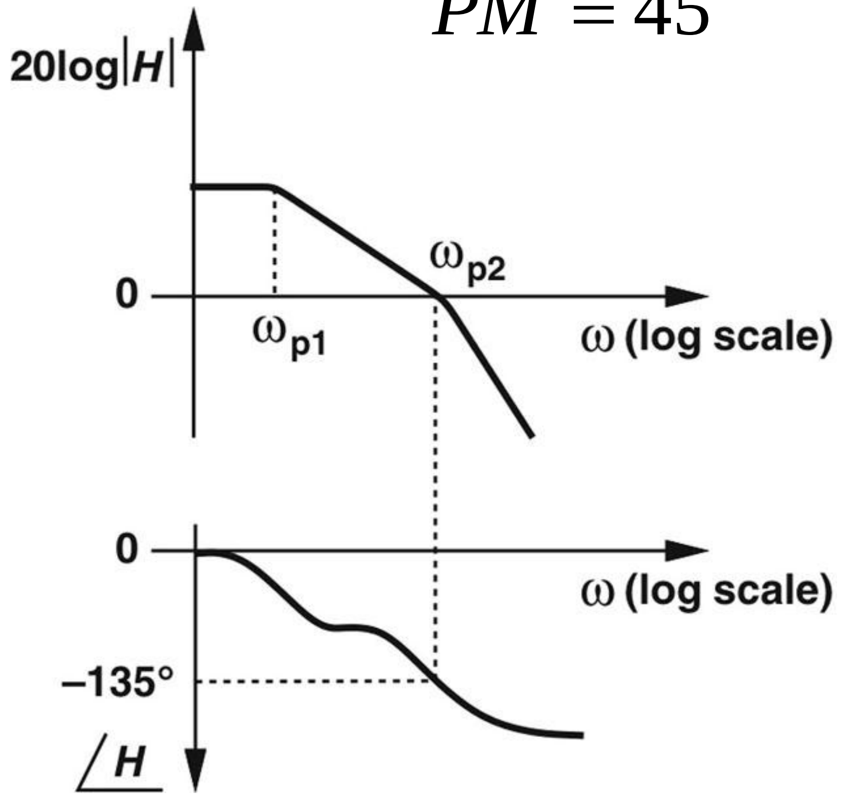


Phase Margin = $\angle H(\omega_{GX})+180$

- **The larger the phase margin, the more stable the negative feedback becomes.**

Phase Margin Example

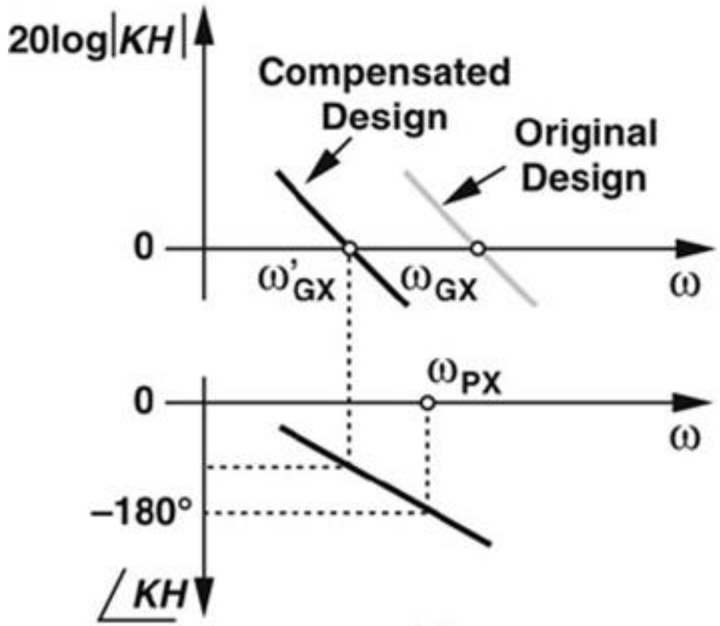
$$PM = 45^\circ$$



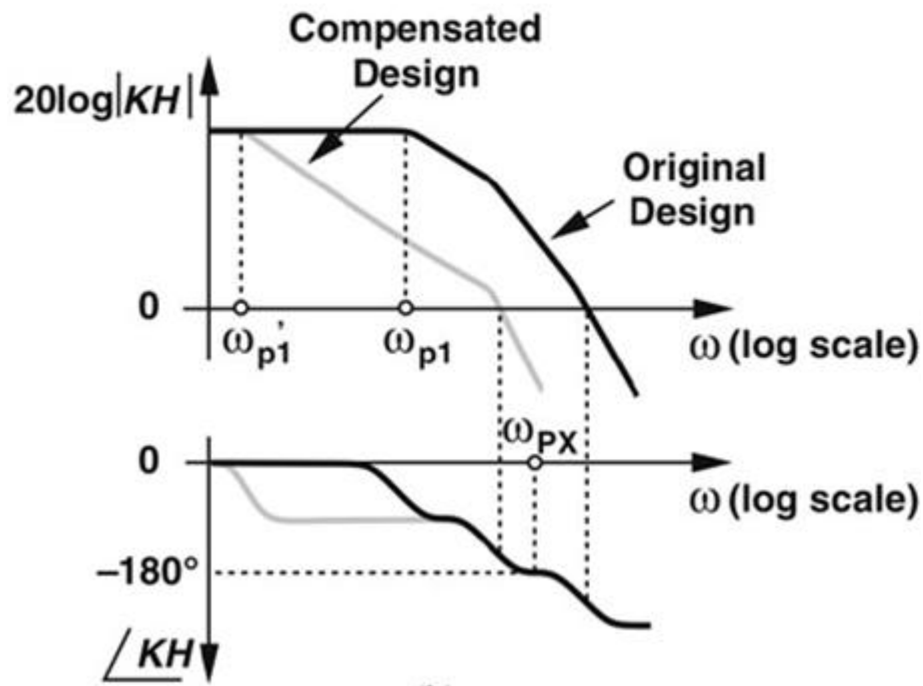
- **For a well-behaved response, a phase margin of 60° is required.**

Frequency Compensation

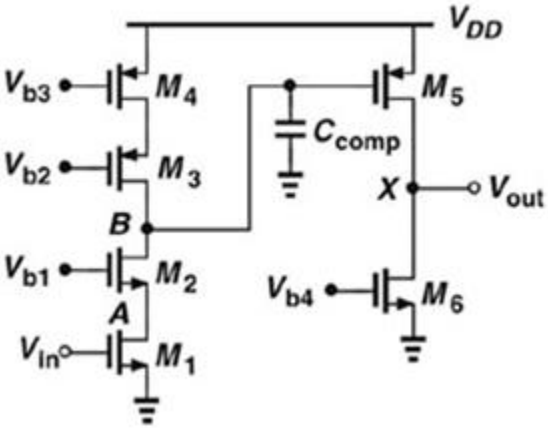
Shift ω_{GX} toward the origin



Shift ω_{P1} to ω_{P1}'



Frequency Compensation Example



$$\omega_{p,A} \approx \frac{g_{m2}}{C_A}$$

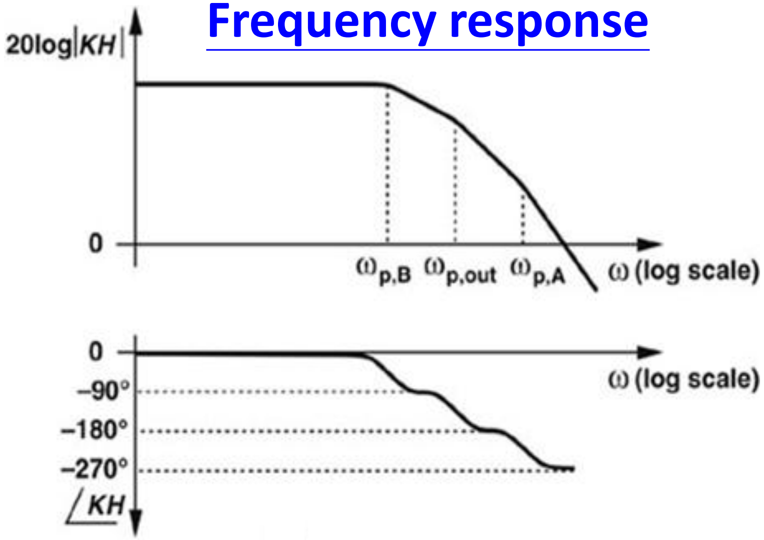
$$\omega_{p,B} \approx \frac{1}{[(g_{m2}r_{O2}r_{O1})||(g_{m3}r_{O3}r_{O4})]C_B}$$

$$\omega_{p,out} = \frac{1}{(r_{O5}||r_{O6})C_{out}}$$

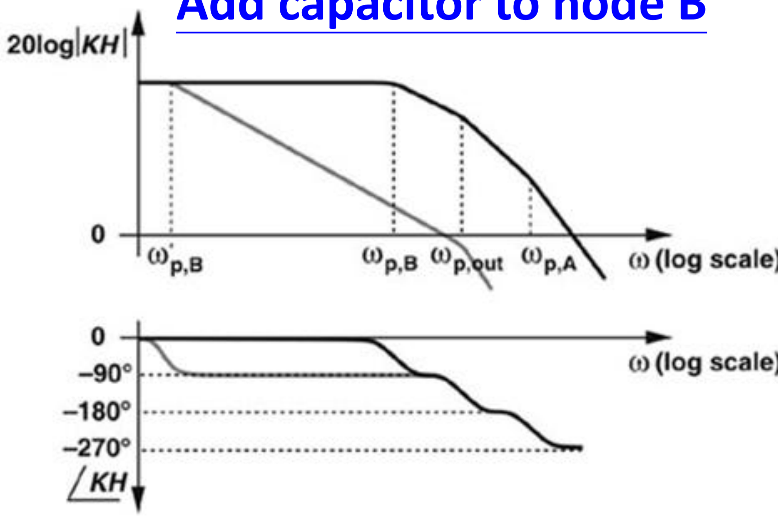
Assume $\omega_{P,B} < \omega_{P,out} < \omega_{P,A}$

Speed degradation!

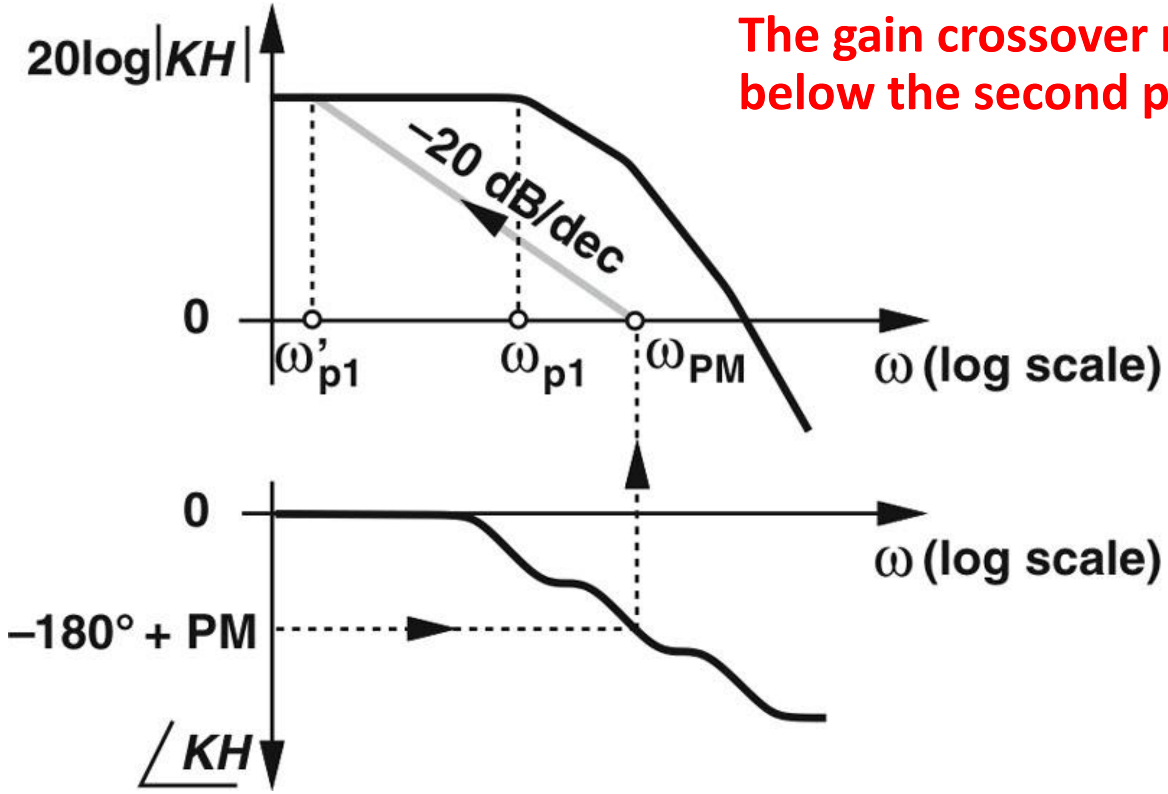
Frequency response



Add capacitor to node B



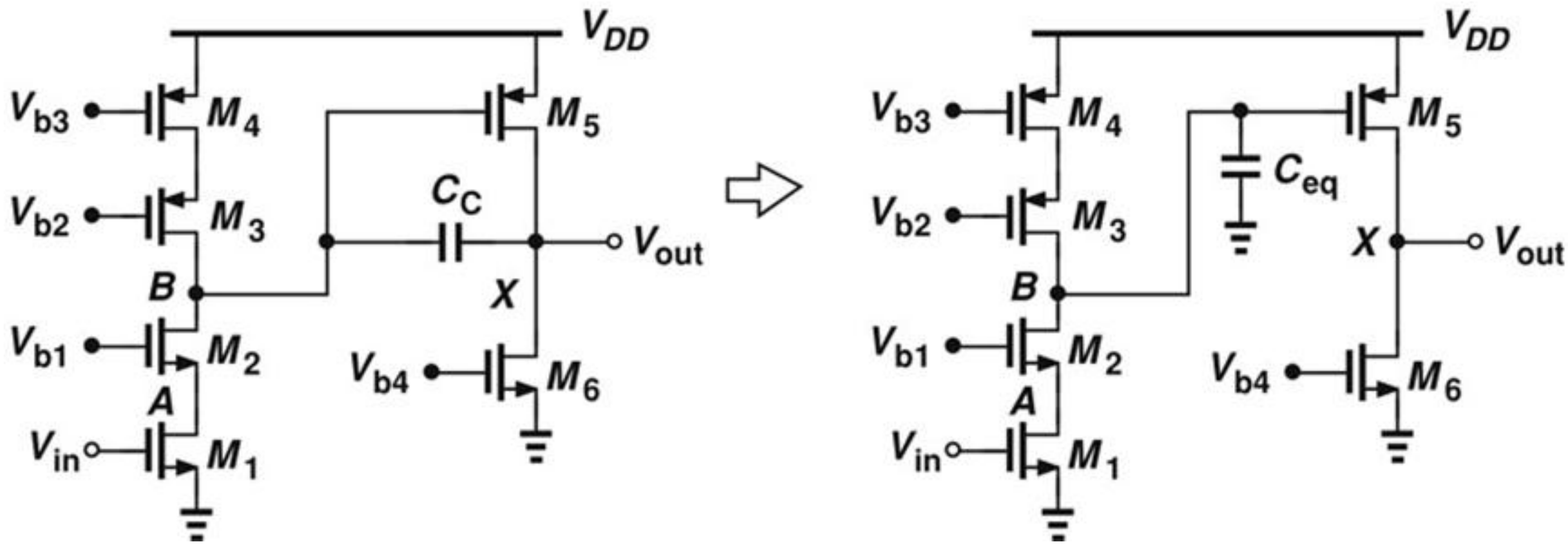
Frequency Response Procedure



The gain crossover must occur below the second pole!!

- We identify a PM, then -180°+PM gives us the new ω_{GX}, or ω_{PM}.
- On the magnitude plot at ω_{PM}, we extrapolate up with a slope of +20dB/dec until we hit the low frequency gain then we look "down" and the frequency we see is our new dominant pole, ω_{P1}'.

Miller Compensation

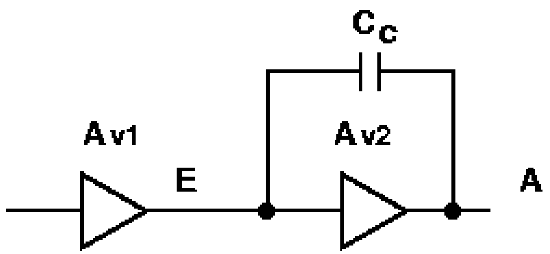


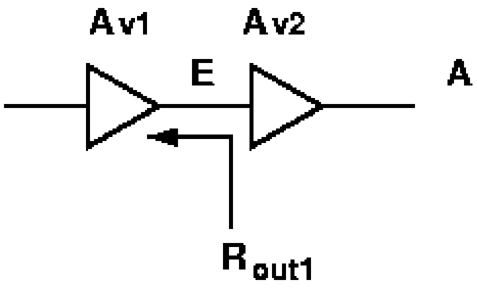
$$C_{eq} = [1 + g_{m5}(r_{O5} \| r_{O6})]C_c$$

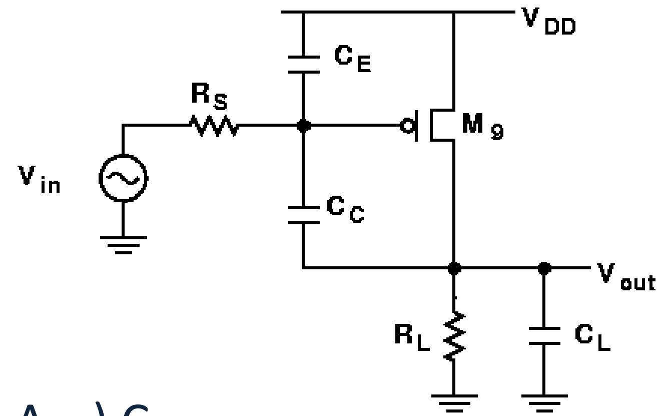
- **To save chip area, Miller multiplication of a smaller capacitance creates an equivalent effect.**

Supporting Materials

Miller Comp. of a Two-Stage Op Amp







- Create a large capacitance at node E, equal to $(1 + A_{v2})\, C_C$.
- The corresponding pole is moved to $1/R_{out1}\left[C_E + \left(1 + A_{v2}\right)C_C\right]$
- Miller compensation
  - Lowering the required capacitor value.
  - Moves the output pole away from the origin.
- If R_S denotes the output resistance of the first stage, $R_L = r_{O9} \,||\, r_{O11}$.

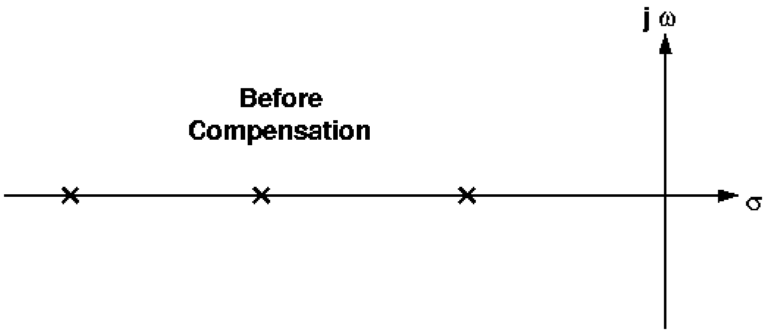
$$\left|\omega_{p1}\right| << \left|\omega_{p2}\right|$$

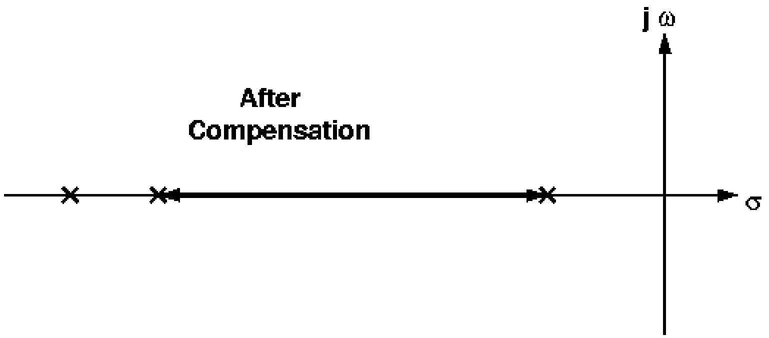
Dominant pole approximation

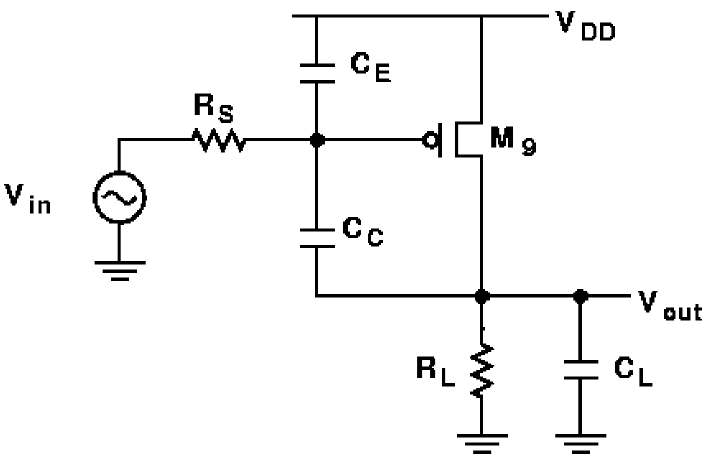
$$\omega_{p1} \approx \frac{1}{R_S\left[\left(1+g_{m9}R_L\right)\left(C_C + C_{GD9}\right) + C_E\right] + R_L\left(C_C + C_{GD9} + C_L\right)}$$

$$\omega_{p2} \approx \frac{R_S\left[\left(1+g_{m9}R_L\right)\left(C_C + C_{GD9}\right) + C_E\right] + R_L\left(C_C + C_{GD9} + C_L\right)}{R_S R_L\left[\left(C_C + C_{GD9}\right)C_E + \left(C_C + C_{GD9}\right)C_L + C_E C_L\right]}$$

Pole Splitting







- Before compensation, ω_{p1} and ω_{p2} are of the same order of magnitude.

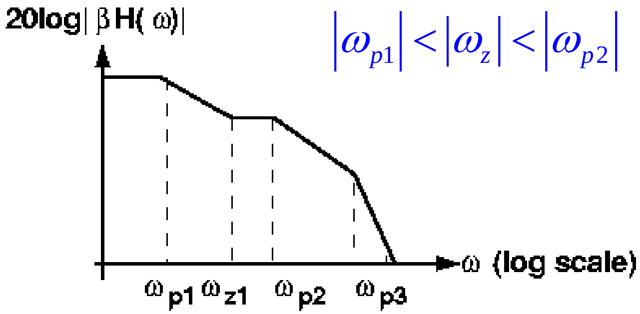
$$C_C = 0, \qquad \omega_{p2} \approx 1/R_L C_L$$

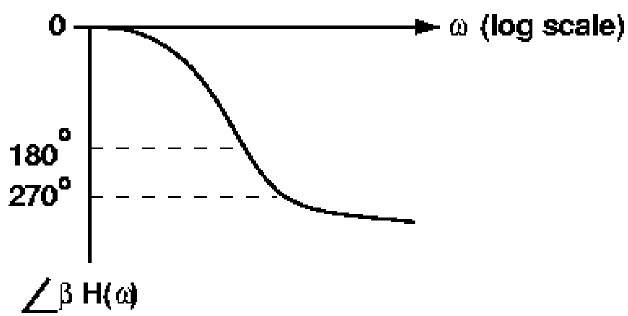
- After compensation ,

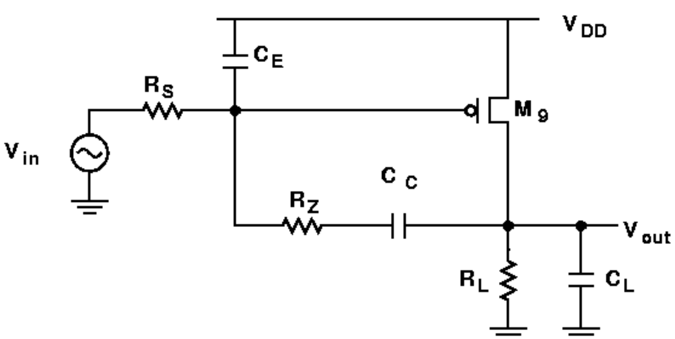
$$\text{If } \; C_C + C_{GD9} >> C_E, \quad \omega_{p2} \approx \frac{g_{m9}}{C_E + C_L} \approx \frac{g_{m9}}{C_L}$$

- **Miller compensation increases the magnitude of the output pole by roughly a factor of $g_{m9} R_L$.**
- **At high frequencies, C_C provides a low impedance between the gate and drain of M_9 . The resistance seen by C_L becomes $R_S \,\|\, g_{m9}^{-1} \,\|\, R_L \approx g_{m9}^{-1}$.**

Effect of Right Half Plane Zero







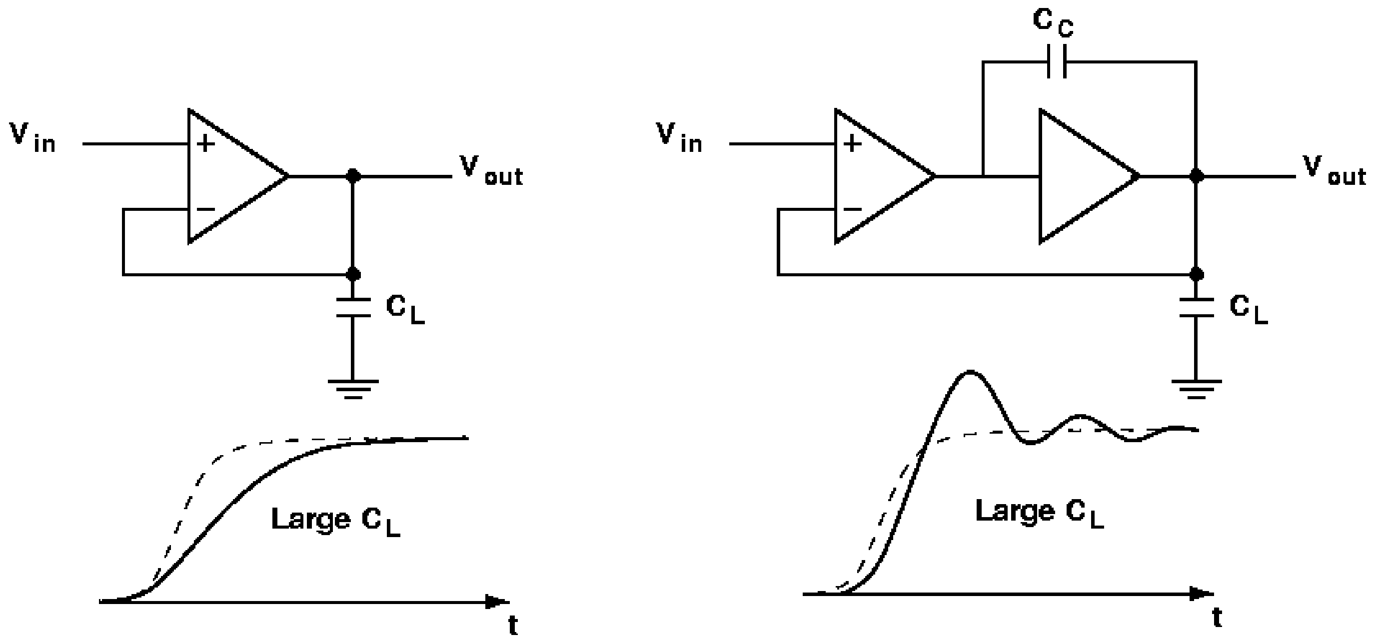
- A right-half plane zero at $\omega_z = g_{m9}/\left(C_C + C_{GD9}\right)$
- Numerator is $\left(1 - s/\omega_z\right)$
- The phase shift is $-\tan^{-1}\left(\omega/\omega_z\right)$ ➔ negative
- Zero slow down the drop of magnitude.
- The stability degrades considerably.
- The right half–plane zero in a two-stage CMOS op amps is a serious issue because gm is relatively small and C_C is usually large.
- A series resistor to eliminate the right-half plane zero.

$$\omega_z \approx \frac{1}{C_C\left(g_{m9}^{-1} - R_Z\right)}$$

- The output stage now exhibits three poles.
- To cancel the first non-dominant pole

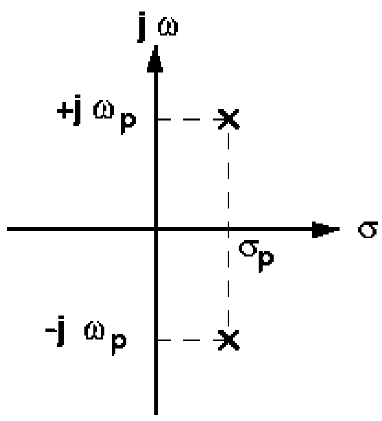
$$\frac{1}{C_C\left(g_{m9}^{-1} - R_Z\right)} = \frac{-g_{m9}}{C_L + C_E}, \quad R_Z = \frac{C_L + C_E + C_C}{g_{m9} C_C} \approx \frac{C_L + C_C}{g_{m9} C_C}$$

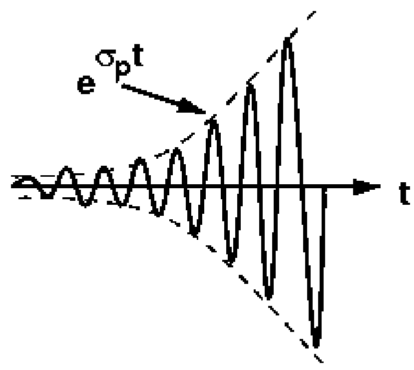
Increased Load Cap. on Step Response

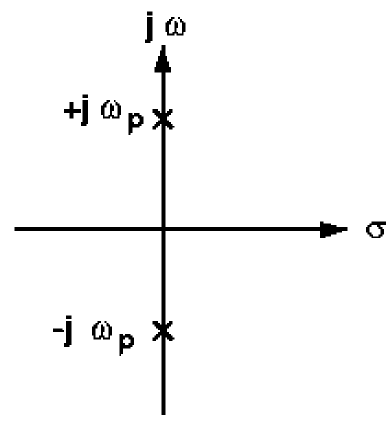


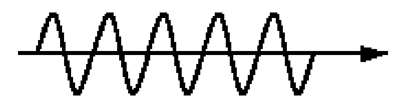
- In a two-stage op-amp, a higher load capacitance presented to the second stage moves the second pole toward the origin, degrading the phase margin.
  – The response approaches an oscillatory behavior if the load capacitance seen by the two-stage op amp increases.
- In one-stage op amps, a higher load capacitance brings the dominant pole closer to the origin, improving the phase margin.
  – Making the feedback system more overdamped.

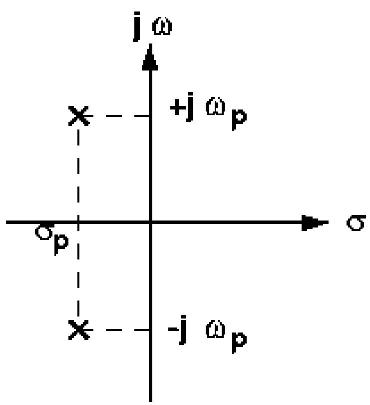
Time Response vs. the Position of Poles

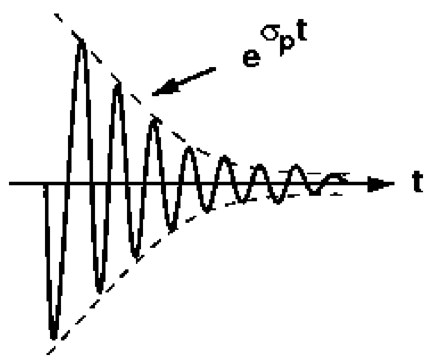












- Expressing each pole frequency as

$$s_p = j\omega_p + \sigma_p$$

- The impulse response of the system includes a term

$$\exp\left(j\omega_p + \sigma_p\right)t$$

Stability and Pole Location

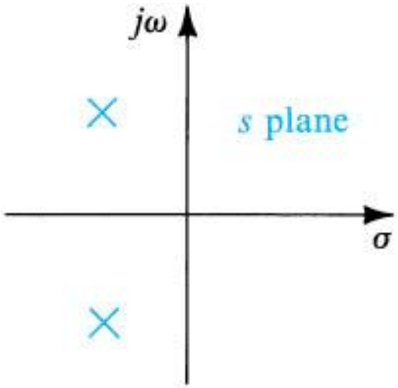
http://en.wikipedia.org/wiki/Laplace_transform

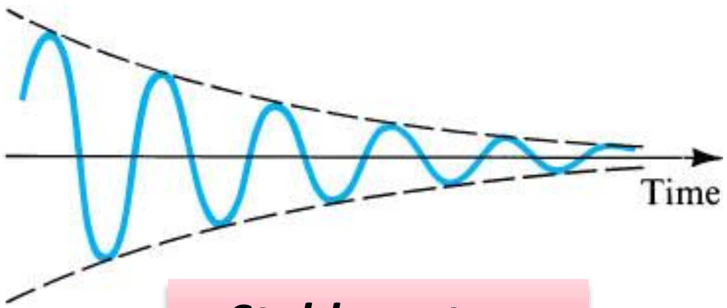
$$H(s) = \frac{1}{(s+\alpha)(s+\beta)}. \qquad h(t) = \mathcal{L}^{-1}\{H(s)\}.$$

Consider an amplifier with a complex-conjugate pole pair $s = \sigma_0 \pm j\omega_n$ ***:***

$$v(t) = e^{\sigma_0 t}[e^{j\omega_n t} + e^{-j\omega_n t}] = 2e^{\sigma_0 t}\cos(\omega_n t)$$

A sinusoidal signal with an envelope $e^{\sigma_0 t}$



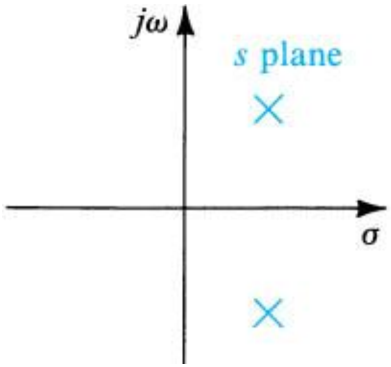


Stable system

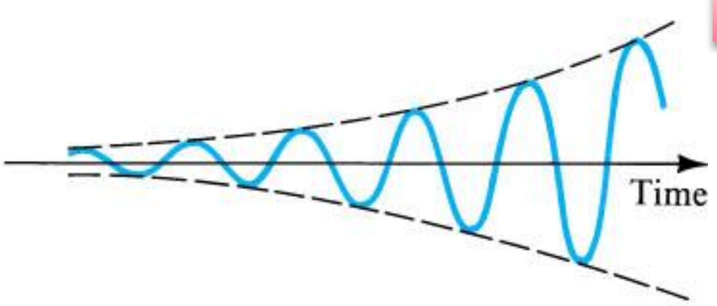
Left-plane pole $\sigma_0 < 0$ ***: Oscillation decays exponentially to zero.***

Stability and Pole Location

Right-plane pole $\sigma_0 > 0$ ***: Oscillation grows exponentially to nonlinear.***

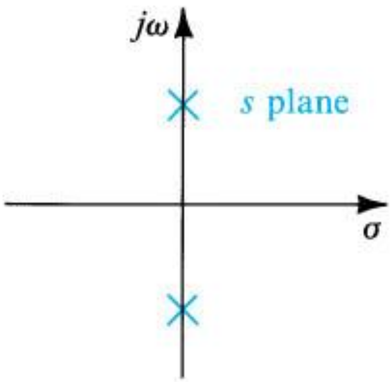


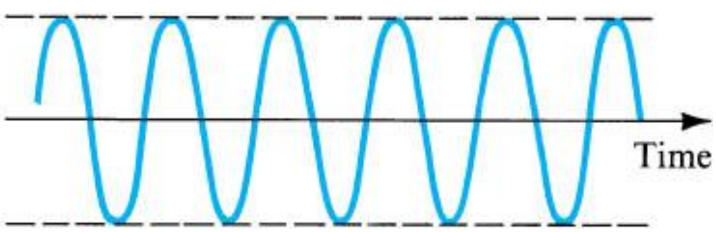
Unstable system



jw axis pole $\sigma_0 = 0$ ***: Oscillation sustained.***

Oscillation system





The existence of any righ-half-plane poles results in instability

Phase Margin

- If $\angle \beta H(j\omega_u) = -175^o, \quad \beta H(j\omega_1) = 1 \times \exp(-j175^o)$

$$\frac{Y}{X}(j\omega_1) = \frac{H(j\omega_1)}{1+\beta H(j\omega_1)} = \frac{\frac{1}{\beta}\exp(-j175^o)}{1+\exp(-j175^o)} = \frac{1}{\beta}\cdot\frac{-0.9962 - j0.0872}{0.0038 - j0.0872}$$

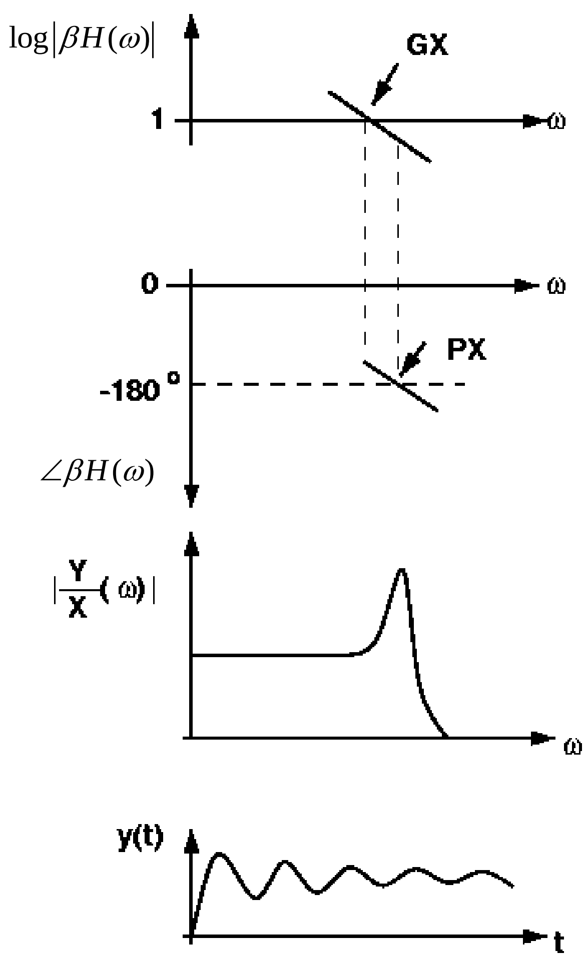
$$\left|\frac{Y}{X}(j\omega_1)\right| = \frac{1}{\beta}\cdot\frac{1}{0.0872} \approx \frac{11.5}{\beta}$$

- Since at low frequencies, $\left| Y/X \right| \approx 1/\beta$, the closed-loop frequency response exhibits a sharp peak in the vicinity of $\omega = \omega_1$

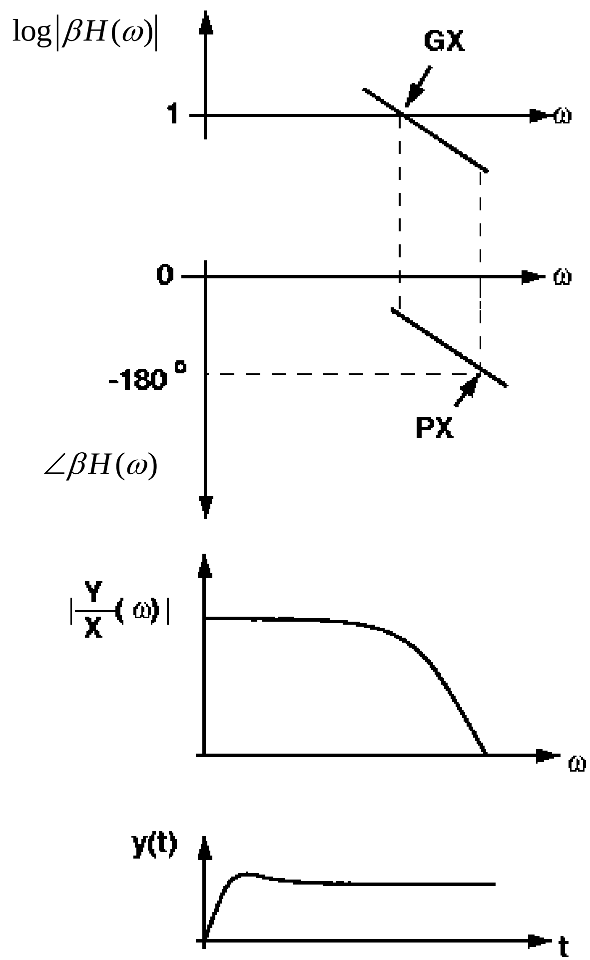
- The closed loop system is near oscillation and its step response exhibits a very underdamped behavior.

Closed-Loop Freq. and Time Response

- **For a small phase margin**



- **For a large phase margin**



- The greater the phase margin, the more stable the feedback system.

CL Freq. Response with 45° PM

- For PM = 45°, $\angle\beta H(\omega_1) = -135^o$ $\quad |\beta H(\omega_1)| = 1$

$$\frac{Y}{X} = \frac{H(j\omega_1)}{1 + 1\times\exp(-j135^o)} = \frac{H(j\omega_1)}{0.29 - 0.71j}$$

$$\left|\frac{Y}{X}\right| = \frac{1}{\beta}\cdot\frac{1}{|0.29-0.71j|} \approx \frac{1.3}{\beta}$$

  – The feedback system suffers from a 30% peak at $\omega = \omega_1$
- For PM = 60°, the peaking is negligible $\left|\frac{Y}{X}\right| = \frac{1}{\beta}$
- The concept of phase margin is well-suited to the design of circuits that process small signals, not large signals.